Annex B

Limit Load Solutions (*Based on SINTAP and R6*)

B Limit Load

B.1 Nomenclature

a	half flaw length for through-thickness flaw, flaw height for surface flaw or half height for embedded flaw
B	specimen thickness, section thickness in plane of flaw
c	half flaw length for surface or embedded flaws
D	diameter
E	Young's modulus
F^N	applied normal load
F_e^N	normal yield limit load
F_e^M	yield limit load for mismatched weldments
F_e^B	yield limit load for base material
H	specimen height
L	pipe or specimen length
L_r^b	normalised limit moment
L_r^N	normalised limit normal load
L_r^p	normalised limit pressure
L_r^{pN}	normalised limit combined pressure and tension load (or n_L ???)
M	mismatch ratio across weldment given by R_e^W/R_e^B
M_{ai}, M_{ao}	applied in and out of plane moments for tubular joints
M_{ci}, M_{co}	fully plastic moments for cracked tubular joints calculated for in and out of plane loads
M^b	applied bending moment
M_e^b	limit bending moment
m^b	applied axisymmetric through wall bending moment per unit angle of cross section
m_e^b	limit axisymmetric through wall bending moment per unit angle of cross section
P_a	applied axial load on tubular joint
P_c	collapse load for cracked tubular joint
P_e	yield limit pressure
p'	Applied pressure
P_e	Limit pressure

r_i inner radius

r_o outer radius

r_m mean radius

R_e Yield stress

R_F^M mismatch flow stress of weldment

R_e^W yield or proof strength of weld metal

R_e^B yield or proof strength of base metal

Q_f factor to allow for the presence of axial and moment loads in the chord.

Q_u strength factor which varies with the joint and load type

t thickness of structural section

t' Effective plate or cylinder thickness for defining local limit load

W Plate width or half plate width

W' Effective half plate width or half cylinder length for defining local limit load

δ crack opening displacement

σ_m membrane stress

σ_b bending stress

$\sigma_{n,m}$ membrane component of collapse stress

$\sigma_{n,b}$ bending component of collapse stress

λ Load ratio for combined tension and bending

ν Poisson's ratio

ϑ crack angle; 90° if perpendicular to the surface

$\alpha,\beta,\eta,\theta,\varphi,\psi,\xi,\zeta$ dimensionless crack size

B.2 Introduction

In classical solid mechanics the limit load is defined as the maximum load a component of elastic-ideally plastic material is able to withstand, above this limit ligament yielding becomes unlimited. In contrast to this definition, real materials strain harden with the consequence that the applied load may increase beyond the value given by the non-hardening limit load. Sometimes strain hardening is roughly taken into account by replacing the yield strength of the material by an equivalent yield strength called 'flow strength' (usually the mean of yield strength and ultimate tensile strength) in the limit load equation.

In a **FITNET FFS analysis** the plastic limit load marks the load at which the plastic zone spreads across the whole ligament ahead of the crack. Some authors prefer the term *yield load* instead of limit load in order to distinguish it from the higher plastic collapse load which is reached when the ligament has completely strain hardened and the component fails under stress controlled loading. The estimation of limit load for a given crack/component geometry is critical input to a fitness-for-service assessment.

This Annex B of the Volume III of the FITNET FFS compiles the K-solutions and limit load solutions along other needed information to conduct FFS analysis. Comprehensive set of limit load solutions are complied to serve as an accurate and user-friendly data. The results of the BS 7910, SINTAP and R6 sources are used to generate this Annex.

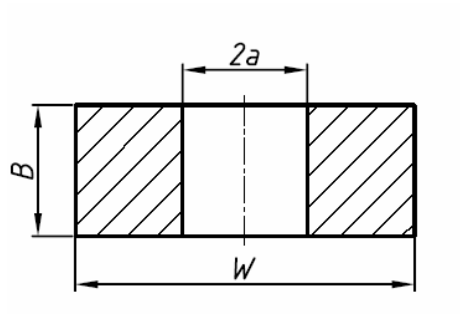
Bending stresses as functions of moments

Structure type	**Bending stress, σ_b with R6 nomenclature**	**Bending stress, σ_b with SINTAP nomenclature**	**Location**
Planar	$\left(\frac{6}{Wt^2}\right)M$	$\left(\frac{6}{wd^2}\right)M$	tensile stress at wall surface (W is plate width)
Pipe with internal circumferential defect (axisymmetric bend)	$\left(\frac{6}{R_m t^2 A_b}\right)m$ $A_b = \frac{12-(t/R_m)^2}{2(6+t/R_m)}$	$\frac{m}{A_b}$ $A_b = \frac{R_1 w^2}{6}\left[2\left(\frac{R_m}{\frac{R_1}{2}+\frac{w}{3}}\right)-3\right]+\frac{w^3}{12}\left[3\left(\frac{R_m}{\frac{R_1}{2}+\frac{w}{3}}\right)-4\right]$	tensile stress at inner wall surface
Pipe with external circumferential defect (axisymmetric bend)	$\left(\frac{6}{R_m t^2 B_b}\right)m$ $B_b = \frac{12-(t/R_m)^2}{2(6-t/R_m)}$	$\frac{m}{B_b}$ $B_b = \frac{R_2 w^2}{6}\left[2\left(\frac{R_m}{\frac{R_2}{2}-\frac{w}{3}}\right)-3\right]+\frac{w^3}{12}\left[4-3\left(\frac{R_m}{\frac{R_1}{2}-\frac{w}{3}}\right)\right]$	tensile stress at outer wall surface
Pipe with internal or external circumferential defect (cantilever bend)	$\left[\frac{2(2+t/R_m)}{\pi R_m^2 t(4+(t/R_m)^2)}\right]M$	$\left[\frac{4R_2}{\pi(R_2^4-R_1^4)}\right]M$	peak tensile stress at outer wall surface

Solid round bar with centrally embedded circular defect (axisymmetric bend)	-	$\left(\frac{192}{w^3}\right)m$	tensile stress at centre of bar
Solid round bar with external circumferential defect (axisymmetric bend)	-	$\left(\frac{96}{w^3}\right)m$	tensile stress at surface of bar
Solid round bar (cantilever bend)	-	$\left(\frac{32}{\pi w^3}\right)M$	peak tensile stress at surface of bar

B.3 Flat plates

B.3.1 Flat plate with through-thickness flaw



R6

Applicable clause(s):

(B.1)

Solution:

$$L_r^N = \frac{F_e^N}{WBR_e}, \quad \beta = \frac{2a}{W}, \quad \gamma = \frac{2}{\sqrt{3}}$$

Plane stress Tresca, plane stress Mises and plane strain Tresca solutions:

$$L_r^N = 1 - \beta \qquad \text{for } 0 \le \beta < 1 \tag{B.1}$$

Plane strain Mises solution:

$$L_r^N = \gamma(1 - \beta) \qquad \text{for } 0 \le \beta < 1 \tag{B.2}$$

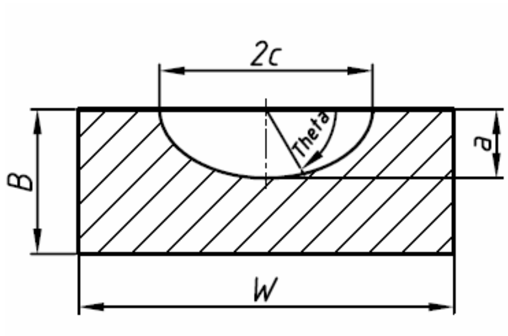
Validity limits:

The plate should be large in comparison to the length of the crack so that edge effects do not influence the results.

Bibliography:

[B.1] A G Miller, Review of limit loads of structures containing defects, Int J Pres Ves Piping **32**, 197-327 (1988).

B.3.2 Flat plate with surface flaw



R6

Plates under combined tension (pin-loaded) and bending:

Applicable clause(s):

– global solution (B.3), (B.4), (B.5), (B.6), (B.7), (B.8)

– local solution (B.9), (B.10), (B.11), (B.12), (B.13), (B.14)

Solution:

Definition:

$$L_r^N = \frac{F_e^N}{WBR_e}, \quad L_r^b = \frac{4M_e^b}{WB^2R_e}, \quad \lambda = \frac{M^b}{BF^N} = \frac{1}{6}\frac{\sigma_b}{\sigma_m}, \; \alpha = \frac{a}{B}, \quad \beta = \frac{2c}{W}, \quad \psi = \frac{c}{B}$$

Global solution:

$$L_r^N = \begin{cases} \dfrac{d_1}{2\lambda + \alpha\beta + \sqrt{(2\lambda + \alpha\beta)^2 + d_1}} & \text{for } \alpha \le \alpha_0 \\ \dfrac{d_2}{2\lambda + \beta\dfrac{1-\alpha}{1-\beta} + \sqrt{\left(2\lambda + \beta\dfrac{1-\alpha}{1-\beta}\right)^2 + \dfrac{d_2}{1-\beta}}} & \text{for } \alpha > \alpha_0 \end{cases} \tag{B.3}$$

$$L_r^b = \begin{cases} \dfrac{4\lambda d_1}{2\lambda + \alpha\beta + \sqrt{(2\lambda + \alpha\beta)^2 + d_1}} & \text{for } \alpha \le \alpha_0 \\ \dfrac{4\lambda d_2}{2\lambda + \beta\dfrac{1-\alpha}{1-\beta} + \sqrt{\left(2\lambda + \beta\dfrac{1-\alpha}{1-\beta}\right)^2 + \dfrac{d_2}{1-\beta}}} & \text{for } \alpha > \alpha_0 \end{cases} \tag{B.4}$$

where

$$d_1 = (1 - \alpha\beta)^2 + 2\alpha^2\beta(1-\beta) \tag{B.5}$$

$$d_2 = (1-\alpha\beta)\left[2-\left(\frac{1-\alpha\beta}{1-\beta}\right)\right]+2\alpha\beta(1-\alpha) \qquad \text{(B.6)}$$

$$\alpha_0 = -\left(\lambda-\frac{1}{2}\right)+\sqrt{\left(\lambda-\frac{1}{2}\right)^2+\frac{\lambda}{1-\frac{1}{2}\beta}} \qquad \text{(B.7)}$$

For pure tension ($\lambda = 0$) and pure bending ($\lambda \to \infty$)

$$\alpha_0 = \begin{cases} 1 & \text{for } \lambda = 0 \\ \dfrac{1}{2-\beta} & \text{for } \lambda \to \infty \end{cases} \qquad \text{(B.8)}$$

Extended surface cracks (set $\beta = 1$): equivalent to the L_r solution formerly in IV.1.8.1.

Through-wall cracks (set $\alpha = 1$)

Local solution (W = B + c and B'= B):

$$L_r^N = \begin{cases} \dfrac{d_1}{2\lambda+\dfrac{\alpha\psi}{1+\psi}+\sqrt{\left(2\lambda+\dfrac{\alpha\psi}{1+\psi}\right)^2+d_1}} & \text{for } \alpha \le \alpha_0 \\ \dfrac{d_2}{2\lambda+\psi(1-\alpha)+\sqrt{(2\lambda+\psi(1-\alpha))^2+(1+\psi)d_2}} & \text{for } \alpha > \alpha_0 \end{cases} \qquad \text{(B.9)}$$

$$L_r^b = \begin{cases} \dfrac{4\lambda d_1}{2\lambda+\dfrac{\alpha\psi}{1+\psi}+\sqrt{\left(2\lambda+\dfrac{\alpha\psi}{1+\psi}\right)^2+d_1}} & \text{for } \alpha \le \alpha_0 \\ \dfrac{4\lambda d_2}{2\lambda+\psi(1-\alpha)+\sqrt{(2\lambda+\psi(1-\alpha))^2+(1+\psi)d_2}} & \text{for } \alpha > \alpha_0 \end{cases} \qquad \text{(B.10)}$$

where

$$d_1 = \left(1-\frac{\alpha\psi}{1+\psi}\right)^2+\frac{2\alpha^2\psi}{(1+\psi)^2} \qquad \text{(B.11)}$$

$$d_2 = \left(1-\frac{\alpha\psi}{1+\psi}\right)\left[1-\psi(1-\alpha)\right]+\frac{2\alpha(1-\alpha)\psi}{1+\psi} \qquad \text{(B.12)}$$

$$\alpha_0 = -\left(\lambda-\frac{1}{2}\right)+\sqrt{\left(\lambda-\frac{1}{2}\right)^2+\frac{2(1+\psi)\lambda}{2+\psi}} \qquad \text{(B.13)}$$

For pure tension ($\lambda = 0$) and pure bending ($\lambda \to \infty$)

$$\alpha_0 = \begin{cases} 1 & \text{for } \lambda = 0 \\ \dfrac{\psi + 1}{\psi + 2} & \text{for } \lambda \to \infty \end{cases} \tag{B.14}$$

Validity limits:

For local case the solutions are limited to

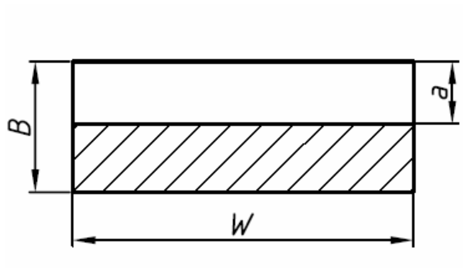
$$\beta < \frac{\psi}{1+\psi}$$

Bibliography:

[B.2] Y Lei, *J*-integral and limit load analysis of semi-elliptical surface cracks in plates under bending, Int J Pres Ves Piping **81**, 34-41 (2004).

[B.3] Y Lei, A global limit load solution for plates with semi-elliptical surface cracks under combined tension and bending, ASME/JSME Pressure Vessels and Piping Conference, San Diego, July 25-29 2004, PVP-Vol. 475, 125-131 (2004).

B.3.3 Flat plate with long surface flaw



Plates under combined tension (pin-loaded) and bending

R6

Applicable clause(s):

(B.15), (B.16)

Solution:

$$L_r^N = \frac{F_e^N}{WBR_e}, \quad L_r^b = \frac{4M_e^b}{WB^2 R_e}, \quad \lambda = \frac{M^b}{BF^N} = \frac{1}{6}\frac{\sigma_b}{\sigma_m}, \quad \alpha = \frac{a}{B}$$

Net-section collapse solution (plane stress Tresca):

$$L_r^N = \frac{(1-\alpha)^2}{2\lambda + \alpha + \sqrt{(2\lambda+\alpha)^2 + (1-\alpha)^2}} \qquad \text{for } \alpha < 1 \qquad \text{(B.15)}$$

$$L_r^b = \frac{4\lambda(1-\alpha)^2}{2\lambda + \alpha + \sqrt{(2\lambda+\alpha)^2 + (1-\alpha)^2}} \qquad \text{for } \alpha < 1 \qquad \text{(B.16)}$$

For the case of pure tension ($\lambda = 0$) eqn. (B.15) applies and for the case of pure bending ($\lambda = \infty$) eqn.(B.16) applies.

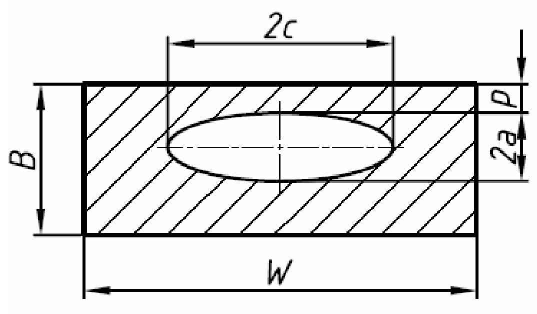
Validity limits:

(The solution is limited to a/B $\leq$ 0.8. Also, the plate should be large in the transverse direction to the crack so that edge effects do not influence the results.)

Bibliography:

[B.4] A A Willoughby and T G Davey, Plastic collapse in part-wall flaws in plates, ASTM STP 1020, American Society for Testing and Materials, Philadelphia, USA, 390-409 (1989).

B.3.4 Flat plate with embedded flaw



Defects in plates under combined tension (pin-loaded) and bending

R6

Applicable clause(s):

IV.1.6.1

Solution:

$$L_r^N = \frac{F_e^N}{WBR_e},\ L_r^b = \frac{4M_e^b}{WB^2R_e},\ \lambda = \frac{M^b}{B\,F^N} = \frac{1}{6}\frac{\sigma_b}{\sigma_m},\ \alpha = \frac{a}{B},\ \beta = \frac{2c}{W},\ k = \frac{\frac{B}{2} - p - a}{B},\ \psi = \frac{c}{B}$$

For symmetrically embedded cracks, k = 0.

Global solution:

Extended embedded cracks set $\beta = 1$

Surface cracks set $\alpha = 0.5 - k$

Through-wall cracks set k = 0 and $\alpha = 0.5$

$$L_r^N = \begin{cases} \dfrac{c_1}{2(\lambda+\alpha\beta)+\sqrt{4(\lambda+\alpha\beta)^2+c_1}} & \text{for } \alpha \le \min(\alpha_1,\alpha_2) \\ \dfrac{c_2}{2[(1-\beta)\lambda+\beta k]+\sqrt{4[(1-\beta)\lambda+\beta k]^2+c_2}} & \text{for } \alpha_1 < \alpha \le \alpha_2 \end{cases} \quad \text{(B.17)}$$

$$L_r^b = \begin{cases} \dfrac{4\lambda c_1}{2(\lambda+\alpha\beta)+\sqrt{4(\lambda+\alpha\beta)^2+c_1}} & \text{for } \alpha \le \min(\alpha_1,\alpha_2) \\ \dfrac{4\lambda c_2}{2[(1-\beta)\lambda+\beta k]+\sqrt{4[(1-\beta)\lambda+\beta k]^2+c_2}} & \text{for } \alpha_1 < \alpha \le \alpha_2 \end{cases} \quad \text{(B.18)}$$

where

$$c_1 = 1 - 8\alpha\beta k - 4(\alpha\beta)^2 \tag{B.19}$$

$$c_2 = (1-\beta)\left(1 - \frac{4\beta k^2}{1-\beta} - 4\beta\alpha^2\right) \tag{B.20}$$

$$\alpha_1 = (k-\lambda)(1-\beta) + \sqrt{(k-\lambda)^2(1-\beta)^2 + \left(\frac{1}{4} - k^2 + 2k\lambda\right)} \tag{B.21}$$

$$\alpha_1 = \begin{cases} k(1-\beta) + \sqrt{\frac{1}{4} - k^2\beta(2-\beta)} & \text{for pure tension } \lambda = 0 \\ \frac{k}{1-\beta} & \text{for pure bending } \lambda \to \infty \end{cases} \tag{B.22}$$

$$\alpha_2 = \frac{1}{2} - k \tag{B.23}$$

Local solution (d = B + c and t_1 = B):

$$\mathrm{L}_r^N = \begin{cases} \dfrac{c_1}{2\left(\lambda + \frac{\alpha\psi}{1+\psi}\right) + \sqrt{4\left(\lambda + \frac{\alpha\psi}{1+\psi}\right)^2 + c_1}} & \text{for } \alpha \le \min(\alpha_1, \alpha_2) \\ \dfrac{c_2}{\frac{2(\lambda + \psi k)}{1+\psi} + \sqrt{4\left(\frac{\lambda + \psi k}{1+\psi}\right)^2 + c_2}} & \text{for } \alpha_1 < \alpha \le \alpha_2 \end{cases} \tag{B.24}$$

$$\mathrm{L}_r^b = \begin{cases} \dfrac{4\lambda c_1}{2\left(\lambda + \frac{\alpha\psi}{1+\psi}\right) + \sqrt{4\left(\lambda + \frac{\alpha\psi}{1+\psi}\right)^2 + c_1}} & \text{for } \alpha \le \min(\alpha_1, \alpha_2) \\ \dfrac{4\lambda c_2}{\frac{2(\lambda + \psi k)}{1+\psi} + \sqrt{4\left(\frac{\lambda + \psi k}{1+\psi}\right)^2 + c_2}} & \text{for } \alpha_1 < \alpha \le \alpha_2 \end{cases} \tag{B.25}$$

where

$$c_1 = 1 - \frac{8\alpha k\psi}{1+\psi} - 4\left(\frac{\alpha\psi}{1+\psi}\right)^2 \tag{B.26}$$

$$c_2 = \frac{1}{1+\psi}\left(1 - 4\psi k^2 - \frac{4\psi\alpha^2}{1+\psi}\right) \tag{B.27}$$

$$\alpha_1 = \frac{k-\lambda}{1+\psi} + \sqrt{\left(\frac{k-\lambda}{1+\psi}\right)^2 + \left(\frac{1}{4} - k^2 + 2k\lambda\right)} \tag{B.28}$$

$$\alpha_1 = \begin{cases} \dfrac{k}{1+\psi} + \sqrt{\dfrac{1}{4} - \dfrac{k^2\psi(2+\psi)}{(1+\psi)^2}} & \text{for pure tension } \lambda = 0 \\ k(1+\psi) & \text{for pure bending } \lambda \to \infty \end{cases} \tag{B.29}$$

$$\alpha_2 = \frac{1}{2} - k \tag{B.30}$$

Global solution (pin-loaded):

Equations (IV.1.6.1-1) and (IV.1.6.1-3) to (IV.1.6.1-7) (set λ = 0). (IV.1.6.3-1)

For embedded extended defects, set β =1 and $\psi = \infty$.

Local solutions (pin-loaded):

For embedded extended defects, set β =1 and $\psi = \infty$.

(a) W'= B + c, B' = B and W/2 > W':

Equations (IV.1.6.2-1) and (IV.1.6.2-3) to (IV.1.6.2-7) (set λ = 0). (IV.1.6.3-2)

(b) $d = B\left(1 - \frac{2\alpha}{1-2k}\right) + c$, $t_1 = B(1-2k)$ and W/2 > d :

$$L_r^N = \frac{\left(1 - \frac{2\alpha}{1-2k}\right)(1+\psi)}{\left(1 - \frac{2\alpha}{1-2k}\right) + \psi} \tag{B.31}$$

Global solution (fixed-grip tension):

Equations (IV.1.6.1-1) and (IV.1.6.1-3) to (IV.1.6.1-7) (set λ = 0 and k = 0 as limit load value does not depend on crack position in the cross section).

For embedded extended defects, set β =1 and $\psi = \infty$.

Local solution (fixed-grip tension):

For W' = B + c, B'= B and W/2 > W':

For embedded extended defects, set β =1 and $\psi = \infty$.

$$L_r^N = \frac{1+\psi(1-2\alpha)}{1+\psi} \tag{B.32}$$

Global solution (bending):

Equations (IV.1.6.1-2) to (IV.1.6.1-7) (set $\lambda = \infty$) (IV.1.6.4-1)

For embedded extended defects, set β =1 and $\psi = \infty$.

Local solutions (bending):

For embedded extended defects, set β =1 and $\psi = \infty$.

(a) W' = B + c, B' = B and W/2 > W':

Equations (IV.1.6.2-2) to (IV.1.6.2-7) (set $\lambda = \infty$) (IV.1.6.4-2)

(b) $d = B\left(1-\frac{2\alpha}{1-2k}\right)+c$, $t_1 = B(1-2k)$ and W/2 > d :

$$L_r^b = (1-2k)^2 \frac{\left(1-\frac{2\alpha}{1-2k}\right)+\psi\left(1-\frac{4\alpha^2}{(1-2k)^2}\right)}{1-\frac{2\alpha}{1-2k}+\psi} \qquad \text{(B.33)}$$

Validity limits:

Global solutions are a net-section collapse solution valid for

$\frac{1}{2} > k \geq 0$ and $\alpha \leq \frac{1}{2} - k$.

Local solutions are valid for

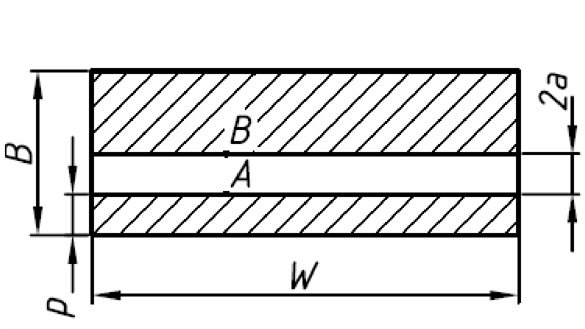
$\frac{1}{2} > k \geq 0$, $\alpha \leq \frac{1}{2} - k$ and $\beta < \frac{\psi}{1+\psi}$

Bibliography:

[B.5] A J Carter, A library of limit loads for FRACTURE-TWO, Nuclear Electric Report TD/SID/REP/0191 (1992).

[B.6] Y Lei and P J Budden, Limit load solutions for plates with embedded cracks under combined tension and bending, Int J Pres Ves Piping 81, 589-597 (2004)

B.3.5 Flat plate with long embedded flaw



R6

See 2.4 when c=W/2

set β = 1 and set $\psi = \infty$

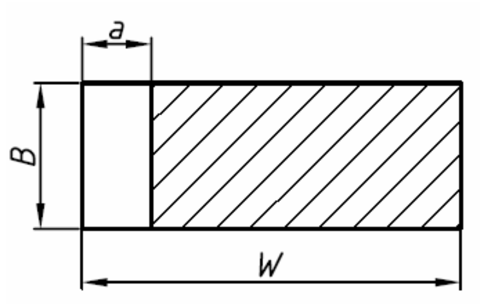
Applicable clause(s):

Solution:

Validity limits:

Bibliography:

B.3.6 Flat plate with edge flaw



R6

Applicable clause(s):

Compact tension specimen (CT) ; Plane Stress & Strain (Mises & Tresca) (B.34) - (B.37)

Three-point-bending specimen (TPB); Plane Strain (Mises & Tresca) (B.38), (B.39)

Single edge cracked plate under tension (SECP); Plane Stress & Strain (Mises & Tresca) (B.40) - (B.45)

Single edge cracked plate under bending (SECB); Plane Stress & Strain (Mises & Tresca) (B.46) - (B.49)

Solution:

Compact tension specimen (CT) ; Plane Stress & Strain (Mises & Tresca)

$$L_r^N = \frac{F_e^N}{WBR_e}, \quad \beta = \frac{a}{W}, \quad \gamma = \frac{2}{\sqrt{3}}$$

Plane stress Tresca solution:

$$L_r^N = \sqrt{2 + 2\beta^2} - (1 + \beta) \qquad \text{for } 0 \le \beta < 1 \tag{B.34}$$

Plane stress Mises solution:

$$L_r^N = \sqrt{(1+\gamma)(1+\gamma\beta^2)} - (1+\gamma\beta) \qquad \text{for } 0 \le \beta < 1 \tag{B.35}$$

Plane strain Tresca solution:

$$L_r^N = \begin{cases} 0.634 - 1.482\beta + 0.134\beta^2 + 0.25\beta^3 & \text{for } 0 \le \beta \le 0.09 \\ \sqrt{2.702 + 4.599\beta^2} - (1 + 1.702\beta) & \text{for } 0.09 < \beta < 1 \end{cases} \tag{B.36}$$

Plane strain Mises solution:

$$L_r^N = \begin{cases} \gamma(0.634 - 1.482\beta + 0.134\beta^2 + 0.25\beta^3) & \text{for } 0 \le \beta \le 0.09 \\ \gamma\left[\sqrt{2.702 + 4.599\beta^2} - (1 + 1.702\beta)\right] & \text{for } 0.09 < \beta < 1 \end{cases} \tag{B.37}$$

Three-point-bending specimen (TPB); Plane Strain (Mises & Tresca)

$$L_r^N = \frac{2LF_e^N}{W^2 BR_e}, \quad \beta = \frac{a}{W}, \quad \gamma = \frac{2}{\sqrt{3}}$$ were L is the loaded length of the specimen

2S: Support distance

Plane strain Tresca solution:

$$L_r^N = \begin{cases} \left(1.12 + 1.13\beta - 3.194\beta^2\right)(1-\beta)^2 & \text{for } 0 \le \beta \le 0.18 \\ 1.22(1-\beta)^2 & \text{for } 0.18 < \beta < 1 \end{cases} \tag{B.38}$$

Plane strain Mises solution:

$$L_r^N = \begin{cases} \gamma\left(1.12 + 1.13\beta - 3.194\beta^2\right)(1-\beta)^2 & \text{for } 0 \le \beta \le 0.18 \\ 1.22\gamma(1-\beta)^2 & \text{for } 0.18 < \beta < 1 \end{cases} \tag{B.39}$$

Single edge cracked plate under tension (SECP); Plane Stress & Strain (Mises & Tresca)

$$L_r^N = \frac{F_e^N}{WBR_e}, \quad \beta = \frac{a}{W}, \quad \gamma = \frac{2}{\sqrt{3}}, \quad \eta = \frac{\gamma - 1}{2}$$

pin-loaded:

Plane stress Tresca solution:

$$L_r^N = \sqrt{(1-\beta)^2 + \beta^2} - \beta \qquad \text{for } 0 \le \beta < 1 \tag{B.40}$$

Plane stress Mises solution:

$$L_r^N = \begin{cases} 1 - \beta - 1.232\beta^2 + \beta^3 & \text{for } 0 \le \beta \le 0.545 \\ 1.702\left[\sqrt{(0.794 - (1-\beta))^2 + 0.5876(1-\beta)^2} - (0.794 - (1-\beta))\right] & \text{for } 0.545 < \beta < 1 \end{cases} \tag{B.41}$$

Plane strain Tresca solution:

$$L_r^N = \begin{cases} \gamma\left[1 - \beta - 1.232\beta^2 + \beta^3\right] & \text{for } 0 \le \beta \le 0.545 \\ 1.702\gamma\left[\sqrt{(0.794 - (1-\beta))^2 + 0.5876(1-\beta)^2} - (0.794 - (1-\beta))\right] & \text{for } 0.545 < \beta < 1 \end{cases} \tag{B.42}$$

Plane strain Mises solution:

$$n_L = \begin{cases} \gamma\left[1 - \beta - 1.232\beta^2 + \beta^3\right] & \text{for } 0 \le \beta \le 0.545 \\ 1.702\gamma\left[\sqrt{(0.794 - (1-\beta))^2 + 0.5876(1-\beta)^2} - (0.794 - (1-\beta))\right] & \text{for } 0.545 < \beta < 1 \end{cases} \tag{B.43}$$

Fixed grip:

Plane stress Tresca and Mises, and plane strain Tresca solutions:

$$L_r^N = 1 - \beta \qquad \text{for } 0 \le \beta < 1 \tag{B.44}$$

Plane strain Mises solution:

$$L_r^N = \gamma(1-\beta) \qquad \text{for } 0 \le \beta < 1 \tag{B.45}$$

Single edge cracked plate under bending (SECB); Plane Stress & Strain (Mises & Tresca)

$$L_r^b = \frac{4M_e^b}{BW^2R_e}, \quad \beta = \frac{a}{W}, \quad \gamma = \frac{2}{\sqrt{3}}$$

Plane stress Tresca solution:

$$L_r^b = (1-\beta)^2 \qquad \text{for } 0 \le \beta < 1 \tag{B.46}$$

Plane stress Mises solution:

$$L_r^b = \begin{cases} \left(1+0.934\beta-3.034\beta^2\right)(1-\beta)^2 & \text{for } 0 \le \beta \le 0.154 \\ 1.072(1-\beta)^2 & \text{for } 0.154 < \beta < 1 \end{cases} \tag{B.47}$$

Plane strain Tresca solution:

$$L_r^b = \begin{cases} \left(1+1.686\beta-2.72\beta^2\right)(1-\beta)^2 & \text{for } 0 \le \beta \le 0.295 \\ 1.2606(1-\beta)^2 & \text{for } 0.295 < \beta < 1 \end{cases} \tag{B.48}$$

Plane strain Mises solution:

$$L_r^b = \begin{cases} \gamma\left(1+1.686\beta-2.72\beta^2\right)(1-\beta)^2 & \text{for } 0 \le \beta \le 0.295 \\ 1.2606\gamma(1-\beta)^2 & \text{for } 0.295 < \beta < 1 \end{cases} \tag{B.49}$$

Validity limits:

Bibliography:

[B.7] A G Miller, Review of limit loads of structures containing defects, Int J Pres Ves Piping 32, 197-327 (1988)

[B.8] A J Carter, A library of limit loads for FRACTURE-TWO, Nuclear Electric Report TD/SID/REP/0191 (1992).

B.3.7 Flat plate with double edge flaw

B.3.7.1 Finite width plate

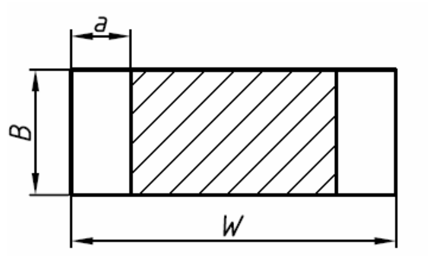


Plate under tension (DECP)

R6

Applicable clause(s):

(B.50) - (B.53)

Solution:

$$L_r^N = \frac{F_e^N}{WBR_e}, \quad \beta = \frac{2a}{W}, \quad \gamma = \frac{2}{\sqrt{3}}$$

Plane stress Tresca solution:

$$L_r^N = 1-\beta \qquad \text{for } 0 \le \beta < 1 \tag{B.50}$$

Plane stress Mises solution:

$$L_r^N = \begin{cases} (1-\beta)(1+0.54\beta) & \text{for } 0 \le \beta \le 0.286 \\ \gamma(1-\beta) & \text{for } 0.286 < \beta < 1 \end{cases} \tag{B.51}$$

Plane strain Tresca solution:

$$L_r^N = \begin{cases} (1-\beta)\left(1+\ln\left(\frac{2-\beta}{2(1-\beta)}\right)\right) & \text{for } 0 \le \beta \le 0.884 \\ 2.57(1-\beta) & \text{for } 0.884 < \beta < 1 \end{cases} \tag{B.52}$$

Plane strain Mises solution:

$$L_r^N = \begin{cases} \gamma(1-\beta)\left(1+\ln\left(\frac{2-\beta}{2(1-\beta)}\right)\right) & \text{for } 0 \le \beta \le 0.884 \\ 2.57\gamma(1-\beta) & \text{for } 0.884 < \beta < 1 \end{cases} \tag{B.53}$$

Validity limits:

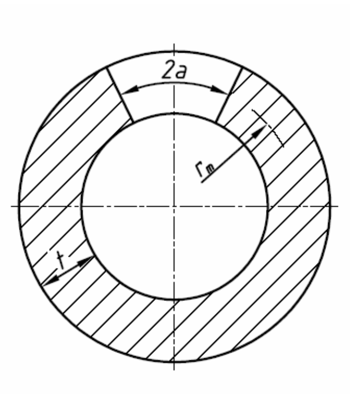
Reference(s):

[B.9] A G Miller, Review of limit loads of structures containing defects, Int J Pres Ves Piping **32**, 197-327 (1988).

B.4 Curved shells

B.4.1 Spheres

B.4.1.1 Through-thickness flaw in a sphere



Membrane stress

R6

Applicable clause(s):

(B.54)

Solution:

$$\eta = \frac{t}{r_m}, \ \theta = \frac{a}{r_m}$$

$$\frac{P_e}{R_e} = \frac{4\eta}{1+\sqrt{1+8\dfrac{\theta^2}{\eta\cos^2\theta}}} \qquad \text{(B.54)}$$

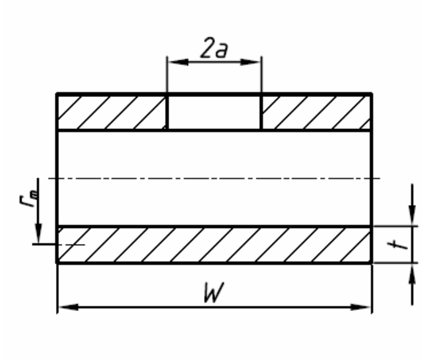
Validity limits:

Bibliography:

[B.10] F M Burdekin and T E Taylor, Fracture in spherical vessels, J Mech Engng Science **11**, 486-497 (1969)

B.5 Pipes or cylinders

B.5.1 Through-thickness cracks in cylinder oriented axially



Membrane stress

R6

Applicable clause(s):

(B.55)

Solution:

$$\eta = \frac{t}{r_m}, \quad \phi = \frac{t}{a}$$

$$\frac{P_e}{R_e} = \frac{\eta}{\sqrt{1+1.05\frac{\eta}{\phi^2}}} \tag{B.55}$$

Validity limits:

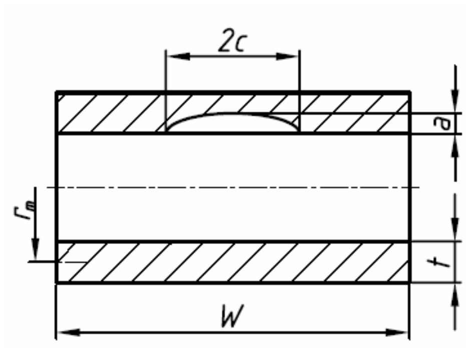
The cylinder should be long in comparison to the length of the crack so that edge effects do not influence the results.

Bibliography:

[B.11] A G Miller, Review of limit loads of structures containing defects, Int J Pres Ves Piping 32, 197-327 (1988).

[B.12] J F Kiefner, W A Maxey, R J Eiber and A R Duffy, Failure stress levels of flaws in pressurised cylinders, ASTM STP 536, American Society for Testing and Materials, Philadelphia, USA, 461-481 (1973).

B.5.2 Internal surface flaw in cylinder oriented axially



Pressure-Excluding or Including Crack Faces; Global & Local Collapse

R6

Applicable clause(s):

(B.56) - (B.60)

Solution:

$$\alpha = \frac{a}{t}, \quad \eta = \frac{t}{r_m}, \quad \phi = \frac{a}{c}$$

(a) Global solutions:

(i) Without defect-face pressure:

$$\frac{P_e}{R_e} = \frac{\alpha\eta}{\left(1-\frac{1}{2}\eta\right)M_g} + \ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) \qquad \text{(B.56)}$$

where

$$M_g = \sqrt{1+1.05\frac{\alpha\eta}{\phi^2\left(1-\frac{1}{2}\eta\right)}} \qquad \text{(B.57)}$$

(ii) With defect-face pressure:

$$\frac{P_e}{R_e} = \frac{\alpha\eta}{\left(1-\frac{1}{2}\eta\right)M_g} + \frac{1-\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) \qquad \text{(B.58)}$$

(b) Local solutions:

(i) Without defect-face pressure ($d = c + s_1(1 - \alpha)$ and $t_1 = B$):

$$\frac{P_e}{R_e} = \frac{s_1(1-\alpha)\ln\left(\dfrac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta}\right) + c\ln\left(\dfrac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right)}{c+s_1(1-\alpha)} \tag{B.59}$$

where

$$s_1 = \frac{\alpha\eta c}{\left(1-\frac{1}{2}\eta\right)M_g \ln\left(1+\dfrac{\alpha\eta}{1-\frac{1}{2}\eta}\right) - \alpha\eta}$$

(ii) With defect-face pressure ($d = c + s_2(1 - \alpha)$ and $t_1 = B$):

$$\frac{P_e}{R_e} = \frac{s_2(1-\alpha)\ln\left(\dfrac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta}\right) + c\dfrac{1-\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\ln\left(\dfrac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right)}{c+s_2(1-\alpha)} \tag{B.60}$$

where

$$s_2 = \frac{\alpha\eta c}{\left(1-\frac{1}{2}\eta\right)M_g \left[\ln\left(\dfrac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta}\right) - \dfrac{1-\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\ln\left(\dfrac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right)\right] - \alpha\eta}$$

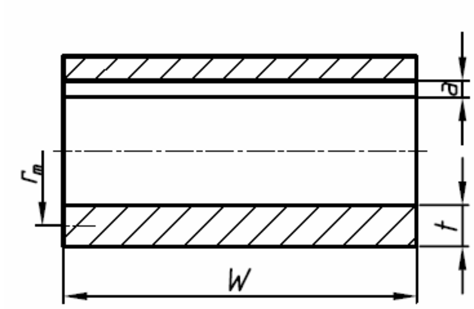
Validity limits:

Bibliography:

[B.13] A G Miller, Review of limit loads of structures containing defects, Int J Pres Ves Piping **32**, 197-327 (1988).

[B.14] A J Carter, A library of limit loads for FRACTURE-TWO, Nuclear Electric Report TD/SID/REP/0191 (1992).

B.5.3 Long internal surface flaw in cylinder oriented axially



Pressure-Excluding or Including Crack Faces

R6

Applicable clause(s):

(B.61), (B.62)

Solution:

$$\alpha = \frac{a}{t}, \qquad \eta = \frac{t}{r_m}$$

Without defect face pressure:

$$\frac{P_e}{R_e} = \ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) \tag{B.61}$$

With defect face pressure:

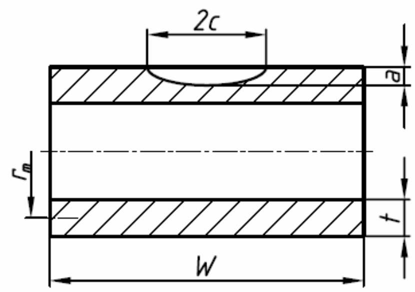
$$\frac{P_e}{R_e} = \frac{1-\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) \tag{B.62}$$

Validity limits:

Bibliography:

[B.15] A J Carter, A library of limit loads for FRACTURE-TWO, Nuclear Electric Report TD/SID/REP/0191 (1992).

B.5.4 External surface flaw in cylinder oriented axially



Membrane and bending stress

R6

Applicable clause(s):

(B.63) - (B.68)

Solution:

$$L_r^N = \frac{\sigma_{n,m}}{R_e},\ L_r^b = \frac{2}{3}\frac{\sigma_{n,b}}{R_e},\ \lambda = \frac{1}{6}\frac{\sigma_b}{\sigma_m},\ \alpha = \frac{a}{t},\ \psi = \frac{c}{t},\ \beta = \frac{2c}{W}$$

Set $\psi = \infty$ for an extended axial external surface crack

Local solutions (W' = t + c and t' = t):

$$L_r^N = \begin{cases} \dfrac{d_1}{2\lambda + \dfrac{\alpha\psi}{1+\psi} + \sqrt{\left(2\lambda + \dfrac{\alpha\psi}{1+\psi}\right)^2 + d_1}} & \text{for } \alpha \le \alpha_0 \\[2ex] \dfrac{d_2}{2\lambda + \psi(1-\alpha) + \sqrt{(2\lambda + \psi(1-\alpha))^2 + (1+\psi)d_2}} & \text{for } \alpha > \alpha_0 \end{cases} \tag{B.63}$$

$$L_r^b = \begin{cases} \dfrac{4\lambda d_1}{2\lambda + \dfrac{\alpha\psi}{1+\psi} + \sqrt{\left(2\lambda + \dfrac{\alpha\psi}{1+\psi}\right)^2 + d_1}} & \text{for } \alpha \le \alpha_0 \\[2ex] \dfrac{4\lambda d_2}{2\lambda + \psi(1-\alpha) + \sqrt{(2\lambda + \psi(1-\alpha))^2 + (1+\psi)d_2}} & \text{for } \alpha > \alpha_0 \end{cases} \tag{B.64}$$

where

$$d_1 = \left(1 - \frac{\alpha\psi}{1+\psi}\right)^2 + \frac{2\alpha^2\psi}{(1+\psi)^2} \tag{B.65}$$

$$d_2 = \left(1 - \frac{\alpha\psi}{1+\psi}\right)[1 - \psi(1-\alpha)] + \frac{2\alpha(1-\alpha)\psi}{1+\psi} \tag{B.66}$$

$$\alpha_0 = -\left(\lambda - \frac{1}{2}\right) + \sqrt{\left(\lambda - \frac{1}{2}\right)^2 + \frac{2(1+\psi)\lambda}{2+\psi}} \tag{B.67}$$

For pure tension ($\lambda = 0$) and pure bending ($\lambda \to \infty$)

$$\alpha_0 = \begin{cases} 1 & \text{for } \lambda = 0 \\ \dfrac{\psi+1}{\psi+2} & \text{for } \lambda \to \infty \end{cases} \tag{B.68}$$

Validity limits:

The solutions are limited to

$$\beta \le \frac{\psi}{1+\psi}$$

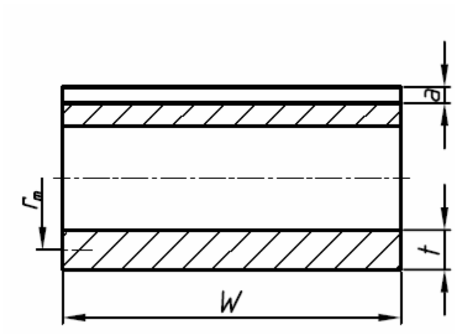
Bibliography:

[B.16] I W Goodall and G A Webster, Theoretical determination of reference stress for partially penetrating flaws in plates, Int J Pres Ves Piping **78,** 687-695 (2001).

[B.17] Y Lei, *J*-integral and limit load analysis of semi-elliptical surface cracks in plates under bending, Int J Pres Ves Piping **81**, 34-41 (2004).

[B.18] Y Lei, A global limit load solution for plates with semi-elliptical surface cracks under combined tension and bending, ASME/JSME Pressure Vessels and Piping Conference, San Diego, July 25-29 2004, PVP-Vol. 475, 125-131 (2004).

B.5.5 Long external surface flaw in cylinder oriented axially



Membrane and bending stress

R6

Applicable clause(s):

IV 1.9.3 with remark VI

„Set ψ = ∞ in the first parts of eqns. (B.63) & (B.64) and eqn. (B.65) to obtain the solution for an extended axial external surface crack in a cylinder under membrane and bending stresses."

Solution:

Validity limits:

Bibliography:

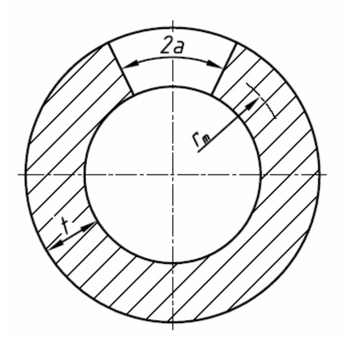
[B.19] I W Goodall and G A Webster, Theoretical determination of reference stress for partially penetrating flaws in plates, Int J Pres Ves Piping **78,** 687-695 (2001).

[B.20] Y Lei, *J*-integral and limit load analysis of semi-elliptical surface cracks in plates under bending, Int J Pres Ves Piping **81**, 34-41 (2004).

[B.21] Y Lei, A global limit load solution for plates with semi-elliptical surface cracks under combined tension and bending, ASME/JSME Pressure Vessels and Piping Conference, San Diego, July 25-29 2004, PVP-Vol. 475, 125-131 (2004).

B.6 Pipes or cylinders with circumferential flaws

B.6.1 Through-thickness flaw in cylinder oriented circumferentially



Membrane and bending stress, global and local solution

R6

Applicable clause(s):

Thick-walled cylinders under combined tension and bending: (B.77)

Thin-walled cylinders under combined tension and bending with internal pressure: (B.72)

"For through-wall defects, $\alpha \equiv 1$"

Solution:

Thick-walled cylinders under combined tension and bending

$$L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e},\; L_r^b = \frac{M_e^b}{4 r_m^2 t R_e},\; \alpha = 1,\; \eta = \frac{t}{r_m},\; \theta = \frac{a}{r_m},\; \lambda = \frac{M^b}{r_m F^N} = \frac{L_r^b}{\frac{\pi}{2} L_r^N}$$

Global solutions:

Whole crack inside the tensile stress zone ($\theta + \beta \le \pi$):

$$\frac{\beta}{\pi} = \frac{1}{2}\left(1 - \frac{\theta}{\pi} - L_r^N\right)$$

$$L_r^b = f_b(\eta)\sin\beta - \frac{1}{2} f_c(\eta)\sin\theta$$

$$f_b = 1 + \frac{1}{12}\eta^2$$

$$f_c = 1 + \frac{1}{6}\eta^2$$

Thin-walled cylinders under combined tension and bending with internal pressure:

$$\alpha = 1,\ L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e},\ L_r^b = \frac{M_e^b}{4 r_m^2 t R_e},\ \theta = \frac{c}{r_m - \frac{t}{2}}$$

$$L_r^p = \frac{\left(r_m - \frac{t}{2}\right)^2 P_e}{2 r_m t R_e} \approx \frac{r_m P_e}{2tR_e} \quad \chi = \frac{F^N}{\pi r_m^2 p'} = \frac{L_r^N}{L_r^p} \quad L_r^{pN} = L_r^p + L_r^N = (1+\chi) L_r^p \tag{B.69}$$

$$\lambda = \frac{M^b}{r_m\left(F^N + \pi r_m^2 p'\right)} = \frac{L_r^b}{\frac{\pi}{2} L_r^{pN}} \tag{B.70}$$

$$S_{a1} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} + \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right) \tag{B.71}$$

$$S_{a2} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} - \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right) \tag{B.72}$$

Global solutions:

Whole crack inside the tensile stress zone ($\theta + \beta \leq \pi$)

$$\frac{\beta}{\pi} = \frac{S_{a1}}{S_{a1} - S_{a2}}\left(1 - \frac{\theta}{\pi} - \frac{L_r^{pN}}{S_{a1}}\right) \tag{B.73}$$

$$L_r^b = \frac{1}{2}\left[\left(S_{a1} - S_{a2}\right)\sin\beta - S_{a1}\sin\theta\right] \tag{B.74}$$

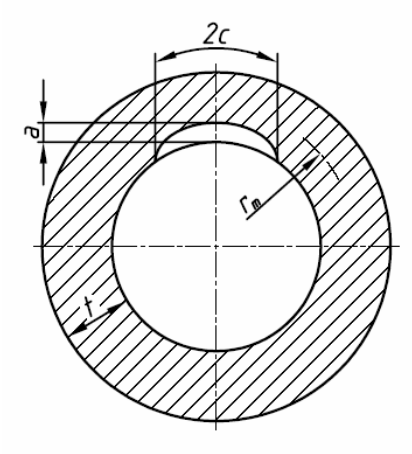
Validity limits:

Bibliography:

[B.22] M R Jones and J M Eshelby, Limit solutions for circumferentially cracked cylinders under internal pressure and combined tension and bending, Nuclear Electric Report TD/SID/REP/0032 (1990).

[B.23] Y Lei and P J Budden, Limit load solutions for thin-walled cylinders with circumferential cracks under combined internal pressure, axial tension and bending, J Strain Analysis 39, 673-683 (2004).

B.6.2 Internal surface flaw in cylinder oriented circumferentially



R6

Applicable clause(s):

Thick-walled cylinders under combined tension and bending: (B.75) - (B.84)

Thin-walled cylinders under combined tension and bending with internal pressure: (B.85) - (B.91)

Solution:

Thick-walled cylinders under combined tension and bending:

$$L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e}, \quad L_r^b = \frac{M_e^b}{4r_m^2 t R_e}, \quad \alpha = \frac{a}{t}, \quad \eta = \frac{t}{r_m}, \ \theta = \frac{c}{r_m - \frac{t}{2}}$$

$$\lambda = \frac{M^b}{r_m F^N} = \frac{L_r^b}{\frac{\pi}{2} L_r^N} \tag{B.75}$$

For through-wall defects, $\alpha \equiv 1$ and for fully circumferential defects, $\theta \equiv \pi$.
Global solutions:

Whole crack inside the tensile stress zone ($\theta + \beta \le \pi$):

$$\frac{\beta}{\pi} = \frac{1}{2}\left(1 - f_a(\eta,\alpha)\alpha\frac{\theta}{\pi} - L_r^N\right) \tag{B.76}$$

$$L_r^b = f_b(\eta)\sin\beta - \frac{1}{2}\alpha\, f_c(\eta,\alpha)\sin\theta \tag{B.77}$$

Part of the crack inside the compression zone ($\theta + \beta > \pi$):

$$\frac{\beta}{\pi} = 1 - \frac{1 + L_r^N - \left[1 - f_e(\eta,\alpha)\right]\dfrac{\theta}{\pi}}{2 f_e(\eta,\alpha)} \tag{B.78}$$

$$L_r^b = f_b(\eta)\left[f_d(\eta,\alpha)\sin\beta + \frac{1}{2}\left(1 - f_d(\eta,\alpha)\right)\sin\theta \right] \tag{B.79}$$

In eqns. (B.76) to (B.79)

$$f_a = 1 - \frac{1}{2}\eta + \frac{1}{2}\alpha\eta \tag{B.80}$$

$$f_b = 1 + \frac{1}{12}\eta^2 \tag{B.81}$$

$$f_c = 1 - \eta + \frac{1}{4}\eta^2 + \alpha\eta - \frac{1}{2}\alpha\eta^2 + \frac{1}{3}\alpha^2\eta^2 \tag{B.82}$$

$$f_d = (1-\alpha)\left[1 + \alpha\eta - \frac{1}{6}\alpha\eta^2 + \frac{1}{3}\alpha^2\eta^2 + \frac{1}{12}\eta^2\right] \Big/ f_b(\eta) \tag{B.83}$$

$$f_e = 1 - \alpha + \frac{1}{2}\alpha\eta - \frac{1}{2}\alpha^2\eta \tag{B.84}$$

Thin-walled cylinders under combined tension and bending with internal pressure:

$$\alpha = \frac{a}{t}, \quad L_r^b = \frac{M_e^b}{4r_m^2 t R_e}, \quad L_r^N = \frac{F_e^N}{2\pi r_m t R_e}, \quad \theta = \frac{c}{r_m - \dfrac{t}{2}}$$

$\alpha \equiv 1$ and for a fully circumferential defect $\theta \equiv \pi$

$$L_r^p = \frac{\left(r_m - \dfrac{t}{2}\right)^2 P_e}{2 r_m t R_e} \approx \frac{r_m P_e}{2 t R_e}$$

$$\chi = \frac{F^N}{\pi \cdot r_m^2 p'} = \frac{L_r^N}{L_r^p}$$

$$L_r^{pN} = L_r^p + L_r^N = (1+\chi)L_r^p$$

$$\lambda = \frac{M^b}{r_m\left(F^N + \pi \cdot r_m^2 p'\right)} = \frac{L_r^b}{\dfrac{\pi}{2} L_r^{pN}} \tag{B.85}$$

$$S_{a1} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} + \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right) \tag{B.86}$$

$$S_{a2} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} - \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right) \tag{B.87}$$

Global solutions:

Whole crack inside the tensile stress zone ($\theta + \beta \leq \pi$)

$$\frac{\beta}{\pi} = \frac{S_{a1}}{S_{a1} - S_{a2}}\left(1 - \alpha\frac{\theta}{\pi} - \frac{L_r^{pN}}{S_{a1}}\right) \tag{B.88}$$

$$L_r^b = \frac{1}{2}\left[(S_{a1} - S_{a2})\sin\beta - S_{a1}\alpha\sin\theta\right] \tag{B.89}$$

Part of the crack inside the compression zone ($\theta + \beta > \pi$)

$$\frac{\beta}{\pi} = \frac{1}{(S_{a1} - S_{a2})(1-\alpha)}\left(S_{a1} - (S_{a1} - S_{a2})\alpha - S_{a2}\,\alpha\frac{\theta}{\pi} - L_r^{pN}\right) \tag{B.90}$$

$$L_r^b = \frac{1}{2}\left[(S_{a1} - S_{a2})(1-\alpha)\sin\beta - S_{a2}\alpha\sin\theta\right] \tag{B.91}$$

Validity limits:

This is a net-section collapse solution. When $\theta + \beta > \pi$, crack closure is ignored. For the cases of combined pressure and bending, this solution may be used by converting the pressure into an equivalent axial load N. However, for very shallow defects, this treatment may overestimate the limit load as the pressure induced hoop stress was ignored in the derivation of this solution.

Bibliography:

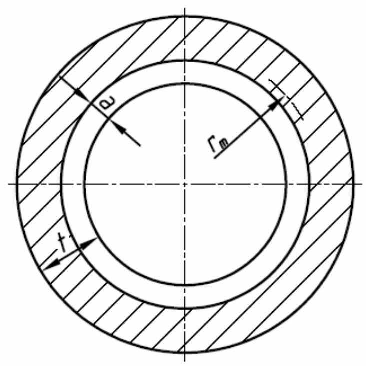
Thick-walled cylinders under combined tension and bending:

[B.24] M R Jones and J M Eshelby, Limit solutions for circumferentially cracked cylinders under internal pressure and combined tension and bending, Nuclear Electric Report TD/SID/REP/0032 (1990).

Thin-walled cylinders under combined tension and bending with internal pressure:

[B.25] Y Lei and P J Budden, Limit load solutions for thin-walled cylinders with circumferential cracks under combined internal pressure, axial tension and bending, J Strain Analysis 39, 673-683 (2004).

B.6.3 Long internal surface flaw in cylinder oriented circumferentially



R6

Applicable clause(s):

Thick-walled cylinders under combined tension and bending:

IV 1.8.1 with remark III "for a fully circumferential defect $\theta \equiv \pi$"
Thick Pipe under internal pressure:

IV 1.8.2

Thin-walled cylinders under combined tension and bending with internal pressure:

IV 1.8.4 with remark III "for a fully circumferential defect $\theta \equiv \pi$"
Thin-walled Cylinder under axial load:
IV 1.8.5

Solution:

Thick-walled cylinders under combined tension and bending:

$$L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e}, \quad L_r^b = \frac{M_e^b}{4 r_m^2 t R_e}, \quad \alpha = \frac{a}{t}, \quad \eta = \frac{t}{r_m}, \ \theta = \pi$$

$$\lambda = \frac{M^b}{r_m F^N} = \frac{L_r^b}{\frac{\pi}{2} L_r^N} \tag{B.92}$$

Global solutions:

$$\frac{\beta}{\pi} = 1 - \frac{1 + L_r^N - [1 - f_e(\eta,\alpha)]}{2 f_e(\eta,\alpha)} \tag{B.93}$$

$$L_r^b = f_b(\eta)[f_d(\eta,\alpha)\sin\beta\ ] \tag{B.94}$$

In eqns. (IV.1.8.1-2) to (IV.1.8.1-5)

$$f_a = 1 - \frac{1}{2}\eta + \frac{1}{2}\alpha\eta \tag{B.95}$$

$$f_b = 1 + \frac{1}{12}\eta^2 \tag{B.96}$$

$$f_c = 1 - \eta + \frac{1}{4}\eta^2 + \alpha\eta - \frac{1}{2}\alpha\eta^2 + \frac{1}{3}\alpha^2\eta^2 \tag{B.97}$$

$$f_d = (1-\alpha)\left[1 + \alpha\eta - \frac{1}{6}\alpha\eta^2 + \frac{1}{3}\alpha^2\eta^2 + \frac{1}{12}\eta^2\right] \Big/ f_b(\eta) \tag{B.98}$$

$$f_e = 1 - \alpha + \frac{1}{2}\alpha\eta - \frac{1}{2}\alpha^2\eta \tag{B.99}$$

Thick Pipe under internal pressure:

$$\alpha = \frac{a}{t}, \qquad \eta = \frac{t}{r_m}$$

With defect face pressure:

$$\frac{P_e}{R_e} = \ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) + \frac{1}{2}\left[\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right)^2 - 1\right] \tag{B.100}$$

if $\quad \dfrac{\alpha\eta}{1-\frac{1}{2}\eta+\alpha\eta} > \dfrac{1}{2}\left[\left(\dfrac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right)^2 - 1\right]$

otherwise

$$\frac{P_e}{R_e} = \ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) + 1 - \frac{1-\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta} \tag{B.101}$$

Without defect face pressure (sealed defect):

$$\frac{P_e}{R_e} = \left(\frac{1-\frac{1}{2}\eta+\alpha\eta}{1-\frac{1}{2}\eta}\right)^2 \ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) + \frac{1}{2}\frac{\eta(1-\alpha)(2+\alpha\eta)}{\left(1-\frac{1}{2}\eta\right)^2} \tag{B.102}$$

if $$\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta}\right) > \left(\frac{1-\frac{1}{2}\eta+\alpha\eta}{1-\frac{1}{2}\eta}\right)^2 \ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) + \frac{1}{2}\frac{\eta(1-\alpha)(2+\alpha\eta)}{\left(1-\frac{1}{2}\eta\right)^2}$$

otherwise

$$\frac{P_e}{R_e} = \ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta}\right) \qquad \text{(B.103)}$$

Thin-walled Cylinder:

$$L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e}, \qquad \alpha = \frac{a}{t}$$

$$L_r^N = \begin{cases} \frac{1}{2}\left(\alpha + \sqrt{(4-\alpha)^2 - 12}\right) & \text{for } \alpha \le \frac{1}{1+\sqrt{3}} \\ \frac{2}{\sqrt{3}}(1-\alpha) & \text{for } \alpha > \frac{1}{1+\sqrt{3}} \end{cases}$$

Thin-walled cylinders under combined tension and bending with internal pressure:

$$\alpha = \frac{a}{t}, \qquad L_r^b = \frac{M_e^b}{4 r_m^2 t R_e}, \qquad L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e}, \ \theta = \pi$$

$$L_r^p = \frac{\left(r_m - \frac{t}{2}\right)^2 P_e}{2 r_m t R_e} \approx \frac{r_m P_e}{2 t R_e}$$

$$\chi = \frac{F^N}{\pi \cdot r_m^2 p'} = \frac{L_r^N}{L_r^p}$$

$$L_r^{pN} = L_r^p + L_r^N = (1+\chi) L_r^p$$

$$\lambda = \frac{M^b}{r_m\left(F^N + \pi \cdot r_m^2 p'\right)} = \frac{L_r^b}{\frac{\pi}{2} L_r^{pN}} \qquad \text{(B.104)}$$

$$S_{a1} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} + \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right) \qquad \text{(B.105)}$$

$$S_{a2} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} - \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2} \right) \quad \text{(B.106)}$$

Global solutions:

$$\frac{\beta}{\pi} = \frac{1}{(S_{a1} - S_{a2})(1-\alpha)}\left(S_{a1} - (S_{a1} - S_{a2})\alpha - S_{a2}\ \alpha - L_r^{pN}\right) \quad \text{(B.107)}$$

$$L_r^b = \frac{1}{2}\left[(S_{a1} - S_{a2})(1-\alpha)\sin\beta\ \right] \quad \text{(B.108)}$$

Thin-walled Cylinder under axial load:

$$L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e}, \qquad \alpha = \frac{a}{t}$$

$$L_r^N = \begin{cases} \frac{1}{2}\left(\alpha + \sqrt{(4-\alpha)^2 - 12}\right) & \text{for } \alpha \le \frac{1}{1+\sqrt{3}} \\ \frac{2}{\sqrt{3}}(1-\alpha) & \text{for } \alpha > \frac{1}{1+\sqrt{3}} \end{cases} \quad \text{(B.109)}$$

Validity limits:

Bibliography:

Thick-walled cylinders under combined tension and bending:

[B.26] M R Jones and J M Eshelby, Limit solutions for circumferentially cracked cylinders under internal pressure and combined tension and bending, Nuclear Electric Report TD/SID/REP/0032 (1990).

Thick Pipe under internal pressure:

[B.27] M R Jones and J M Eshelby, Limit solutions for circumferentially cracked cylinders under internal pressure and combined tension and bending, Nuclear Electric Report TD/SID/REP/0032 (1990).

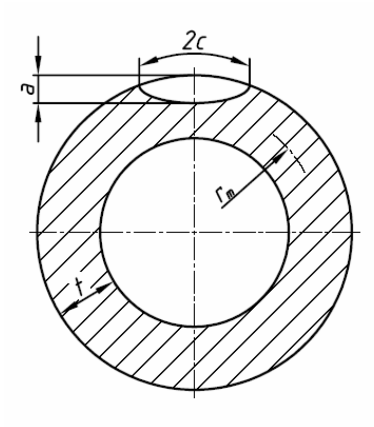
Thin-walled cylinders under combined tension and bending with internal pressure

[B.28] Y Lei and P J Budden, Limit load solutions for thin-walled cylinders with circumferential cracks under combined internal pressure, axial tension and bending, J Strain Analysis **39**, 673-683 (2004).

Thin-walled Cylinder under axial load:

[B.29] R A Ainsworth, Plastic collapse load of a thin-walled cylinder under axial load with a fully circumferential crack, Nuclear Electric Engineering Advice Note EPD/GEN/EAN/0085/98 (1998).

B.6.4 External surface flaw in cylinder oriented circumferentially



R6

Applicable clause(s):

Thick-walled cylinders under combined tension and bending: IV 1.8.1

Thin-walled cylinders under combined tension and bending with internal pressure: IV 1.8.4

Solution:

Thick-walled cylinders under combined tension and bending:

$$L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e}, \quad L_r^b = \frac{M_e^b}{4 r_m^2 t R_e}, \quad \alpha = \frac{a}{t}, \quad \eta = \frac{t}{r_m}, \; \theta = \frac{c}{r_m - \frac{t}{2}}$$

$$\lambda = \frac{M^b}{r_m F^N} = \frac{L_r^b}{\frac{\pi}{2} L_r^N} \tag{B.110}$$

For through-wall defects, $\alpha \equiv 1$ and for fully circumferential defects, $\theta \equiv \pi$.
Global solutions:

Whole crack inside the tensile stress zone ($\theta + \beta \leq \pi$):

$$\frac{\beta}{\pi} = \frac{1}{2}\left(1 - f_a(\eta,\alpha)\alpha\frac{\theta}{\pi} - L_r^N\right) \tag{B.111}$$

$$L_r^b = f_b(\eta)\sin\beta - \frac{1}{2}\alpha\, f_c(\eta,\alpha)\sin\theta \tag{B.112}$$

Part of the crack inside the compression zone ($\theta + \beta > \pi$):

$$\frac{\beta}{\pi} = 1 - \frac{1 + L_r^N - [1 - f_e(\eta,\alpha)]\dfrac{\theta}{\pi}}{2 f_e(\eta,\alpha)} \qquad \text{(B.113)}$$

$$L_r^b = f_b(\eta)\left[f_d(\eta,\alpha)\sin\beta + \frac{1}{2}(1 - f_d(\eta,\alpha))\sin\theta \right] \qquad \text{(B.114)}$$

In eqns. (IV.1.8.1-2) to (IV.1.8.1-5)

$$f_a = 1 + \frac{1}{2}\eta - \frac{1}{2}\alpha\eta \qquad \text{(B.115)}$$

$$f_b = 1 + \frac{1}{12}\eta^2 \qquad \text{(B.116)}$$

$$f_c = 1 + \eta + \frac{1}{4}\eta^2 - \alpha\eta - \frac{1}{2}\alpha\eta^2 + \frac{1}{3}\alpha^2\eta^2 \qquad \text{(B.117)}$$

$$f_d = (1-\alpha)\left[1 - \alpha\eta - \frac{1}{6}\alpha\eta^2 + \frac{1}{3}\alpha^2\eta^2 + \frac{1}{12}\eta^2\right] \Big/ f_b(\eta) \qquad \text{(B.118)}$$

$$f_e = 1 - \alpha - \frac{1}{2}\alpha\eta + \frac{1}{2}\alpha^2\eta \qquad \text{(B.119)}$$

Thin-walled cylinders under combined tension and bending with internal pressure:

$$\alpha = \frac{a}{t}, \quad L_r^b = \frac{M_e^b}{4 r_m^2 t R_e}, \quad L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e}, \ \theta = \frac{c}{r_m + \dfrac{t}{2}}$$

$$L_r^p = \frac{\left(r_m - \dfrac{t}{2}\right)^2 P_e}{2 r_m t R_e} \approx \frac{r_m P_e}{2 t R_e}$$

$$\chi = \frac{F^N}{\pi \cdot r_m^2 p'} = \frac{L_r^N}{L_r^p}$$

$$L_r^{pN} = L_r^p + L_r^N = (1+\chi) L_r^p$$

$$\lambda = \frac{M^b}{r_m (F^N + \pi \cdot r_m^2 p')} = \frac{L_r^b}{\dfrac{\pi}{2} L_r^{pN}} \qquad \text{(B.120)}$$

$$S_{a1} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} + \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2} \right) \qquad \text{(B.121)}$$

$$S_{a2} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} - \sqrt{4-3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right) \tag{B.122}$$

Global solutions:

Whole crack inside the tensile stress zone (θ + β ≤ π)

$$\frac{\beta}{\pi} = \frac{S_{a1}}{S_{a1}-S_{a2}}\left(1-\alpha\frac{\theta}{\pi}-\frac{L_r^{pN}}{S_{a1}}\right) \tag{B.123}$$

$$L_r^b = \frac{1}{2}\left[\left(S_{a1}-S_{a2}\right)\sin\beta - S_{a1}\alpha\sin\theta\right] \tag{B.124}$$

Part of the crack inside the compression zone (θ + β > π)

$$\frac{\beta}{\pi} = \frac{1}{\left(S_{a1}-S_{a2}\right)\left(1-\alpha\right)}\left(S_{a1}-\left(S_{a1}-S_{a2}\right)\alpha - S_{a2}\,\alpha\frac{\theta}{\pi} - L_r^{pN}\right) \tag{B.125}$$

$$L_r^b = \frac{1}{2}\left[\left(S_{a1}-S_{a2}\right)\left(1-\alpha\right)\sin\beta - S_{a2}\alpha\sin\theta\right] \tag{B.126}$$

Validity limits:

Bibliography:

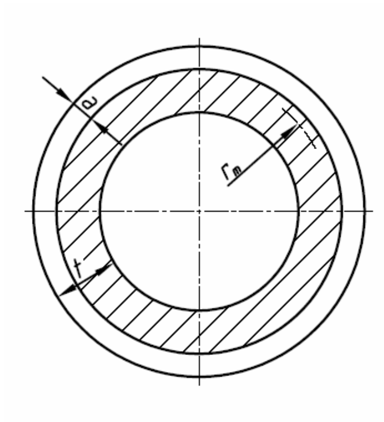
Thick-walled cylinders under combined tension and bending:

[B.30] M R Jones and J M Eshelby, Limit solutions for circumferentially cracked cylinders under internal pressure and combined tension and bending, Nuclear Electric Report TD/SID/REP/0032 (1990).

Thin-walled cylinders under combined tension and bending with internal pressure:

[B.31] Y Lei and P J Budden, Limit load solutions for thin-walled cylinders with circumferential cracks under combined internal pressure, axial tension and bending, J Strain Analysis **39**, 673-683 (2004).

B.6.5 Long external surface flaw in cylinder oriented circumferentially



R6

Applicable clause(s):

Thick-walled cylinders under combined tension and bending:

IV 1.8.1 with remark III “for a fully circumferential defect $\theta \equiv \pi$”
Thick Pipe under internal pressure:

IV 1.8.3

Thin-walled cylinders under combined tension and bending with internal pressure:

IV 1.8.4 with remark III “for a fully circumferential defect $\theta \equiv \pi$”
Thin-walled Cylinder under axial load:
IV 1.8.5

Solution:

Thick-walled cylinders under combined tension and bending:

$$L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e}, \quad L_r^b = \frac{M_e^b}{4 r_m^2 t R_e}, \quad \alpha = \frac{a}{t}, \quad \eta = \frac{t}{r_m}, \ \theta = \pi$$

$$\lambda = \frac{M^b}{r_m F^N} = \frac{L_r^b}{\frac{\pi}{2} L_r^N} \tag{B.127}$$

Global solutions:

$$\frac{\beta}{\pi} = 1 - \frac{1 + L_r^N - [1 - f_e(\eta,\alpha)]}{2 f_e(\eta,\alpha)} \tag{B.128}$$

$$L_r^b = f_b(\eta)[f_d(\eta,\alpha)\sin\beta\ ] \tag{B.129}$$

In eqns. (IV.1.8.1-2) to (IV.1.8.1-5)

$$f_a = 1 + \frac{1}{2}\eta - \frac{1}{2}\alpha\eta \tag{B.130}$$

$$f_b = 1 + \frac{1}{12}\eta^2 \tag{B.131}$$

$$f_c = 1 + \eta + \frac{1}{4}\eta^2 - \alpha\eta - \frac{1}{2}\alpha\eta^2 + \frac{1}{3}\alpha^2\eta^2 \tag{B.132}$$

$$f_d = (1-\alpha)\left[1 - \alpha\eta - \frac{1}{6}\alpha\eta^2 + \frac{1}{3}\alpha^2\eta^2 + \frac{1}{12}\eta^2\right] \Big/ f_b(\eta) \tag{B.133}$$

$$f_e = 1 - \alpha - \frac{1}{2}\alpha\eta + \frac{1}{2}\alpha^2\eta \tag{B.134}$$

Thick Pipe under internal pressure:

$$\alpha = \frac{a}{t}, \quad \eta = \frac{t}{r_m}$$

$$\frac{P_e}{R_e} = \ln\left(\frac{1+\frac{1}{2}\eta - \alpha\eta}{1-\frac{1}{2}\eta}\right) + \frac{1}{2}\left[1 - \left(\frac{1-\frac{1}{2}\eta}{1+\frac{1}{2}\eta-\alpha\eta}\right)^2\right] \tag{B.135}$$

if $\dfrac{1+\frac{1}{2}\eta}{1+\frac{1}{2}\eta-\alpha\eta} > \dfrac{1}{2}\left[1 - \left(\dfrac{1-\frac{1}{2}\eta}{1+\frac{1}{2}\eta-\alpha\eta}\right)^2\right]$

otherwise

$$\frac{P_e}{R_e} = \ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta}\right) \tag{B.136}$$

Thin-walled cylinders under combined tension and bending with internal pressure:

$$\alpha = \frac{a}{t}, \quad L_r^b = \frac{M_e^b}{4r_m^2 tR_e}, \quad L_r^N = \frac{F_e^N}{2\pi \cdot r_m tR_e}, \ \theta = \pi$$

$$L_r^p = \frac{\left(r_m - \frac{t}{2}\right)^2 P_e}{2r_m tR_e} \approx \frac{r_m P_e}{2tR_e}$$

$$\chi = \frac{F^N}{\pi \cdot r_m^2 p'} = \frac{L_r^N}{L_r^p}$$

$$L_r^{pN} = L_r^p + L_r^N = (1+\chi)L_r^p$$

$$\lambda = \frac{M^b}{r_m\left(F^N + \pi \cdot r_m^2 p'\right)} = \frac{L_r^b}{\frac{\pi}{2} L_r^{pN}} \quad \text{(B.137)}$$

$$S_{a1} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} + \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right) \quad \text{(B.138)}$$

$$S_{a2} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} - \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right) \quad \text{(B.139)}$$

Global solutions:

$$\frac{\beta}{\pi} = \frac{1}{(S_{a1} - S_{a2})(1-\alpha)}\left(S_{a1} - (S_{a1} - S_{a2})\alpha - S_{a2}\,\alpha - L_r^{pN}\right) \quad \text{(B.140)}$$

$$L_r^b = \frac{1}{2}\left[(S_{a1} - S_{a2})(1-\alpha)\sin\beta\,\right] \quad \text{(B.141)}$$

Thin-walled Cylinder:

$$L_r^N = \frac{F_e^N}{2\pi \cdot r_m t R_e}, \qquad \alpha = \frac{a}{t}$$

$$L_r^N = \begin{cases} \frac{1}{2}\left(\alpha + \sqrt{(4-\alpha)^2 - 12}\right) & \text{for } \alpha \le \frac{1}{1+\sqrt{3}} \\ \frac{2}{\sqrt{3}}(1-\alpha) & \text{for } \alpha > \frac{1}{1+\sqrt{3}} \end{cases} \quad \text{(B.142)}$$

Validity limits:

Reference(s):

Thick-walled cylinders under combined tension and bending:

[B.32] M R Jones and J M Eshelby, Limit solutions for circumferentially cracked cylinders under internal pressure and combined tension and bending, Nuclear Electric Report TD/SID/REP/0032 (1990).

Thick Pipe under internal pressure:

[B.33] M R Jones and J M Eshelby, Limit solutions for circumferentially cracked cylinders under internal pressure and combined tension and bending, Nuclear Electric Report TD/SID/REP/0032 (1990).

Thin-walled cylinders under combined tension and bending with internal pressure

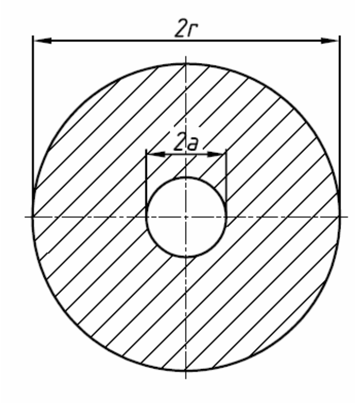
[B.34] Y Lei and P J Budden, Limit load solutions for thin-walled cylinders with circumferential cracks under combined internal pressure, axial tension and bending, J Strain Analysis **39**, 673-683 (2004).

Thin-walled Cylinder under axial load:

[B.35] R A Ainsworth, Plastic collapse load of a thin-walled cylinder under axial load with a fully circumferential crack, Nuclear Electric Engineering Advice Note EPD/GEN/EAN/0085/98 (1998).

B.7 Round bars and bolts

B.7.1 Embedded flaws in round bars



SINTAP

Applicable clause(s):

p. AII. 7-8.

Solution:

Through wall bending for infinite axisymmetric body

Embedded Defect; Through Wall Bending

$$\sigma_{n,b} = \frac{8R_e(\sqrt{2}-1)}{\sqrt{2}}, \quad \sigma_{n,b} = \left(\frac{192}{t^3}\right)m_e^b \tag{B.143}$$

Surface Defect; Through Wall Bending

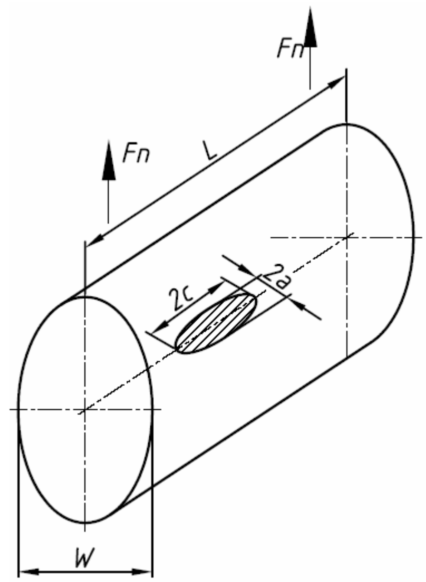
$$\sigma_{n,b} = \frac{8R_e(\sqrt{2}-1)}{\sqrt{2}}, \quad \sigma_{n,b} = \left(\frac{96}{t^3}\right)m_e^b \tag{B.144}$$

Validity limits:

Bibliography:

[B.36] A. J. Carter, A Library of Limit Loads for FRACTURE.TWO, Nuclear Electric Report TD/SID/REP/0191, (1992).

B.7.2 Centrally embedded axial elliptical defects



SINTAP

Applicable clause(s):

p. AII. 38-39

Solution:

Tension; Global & Local Collapse

Global Collapse

$$F_e^N = R_e WL\left(1 - \frac{2ac}{W(W+c)}\right) \tag{B.145}$$

Local Collapse

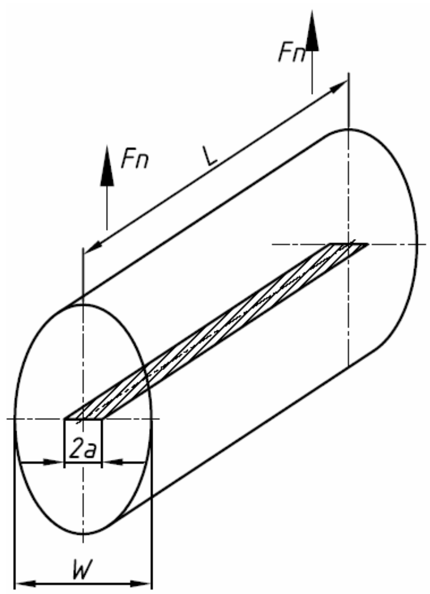
$$F_e^N = R_e WL\left(1 - \frac{2bc}{W(W-2a+c)}\right) \tag{B.146}$$

Validity limits:

Bibliography:

[B.37] A. J. Carter, A Library of Limit Loads for FRACTURE.TWO, Nuclear Electric Report TD/SID/REP/0191, (1992).

B.7.3 Solid round bar; centrally embedded extended defect



SINTAP

Applicable clause(s):

p. AII. 33

Solution:

Radial Tension

$$F_e^N = R_e W L \left(1 - \frac{2a}{W}\right) \quad \text{(B.147)}$$

Validity limits:

Bibliography:

[B.38] A. J. Carter, A Library of Limit Loads for FRACTURE.TWO, Nuclear Electric Report TD/SID/REP/0191, (1992).

B.8 Tubular joints

B.8.1 T- and Y-Joints with axial load

SINTAP

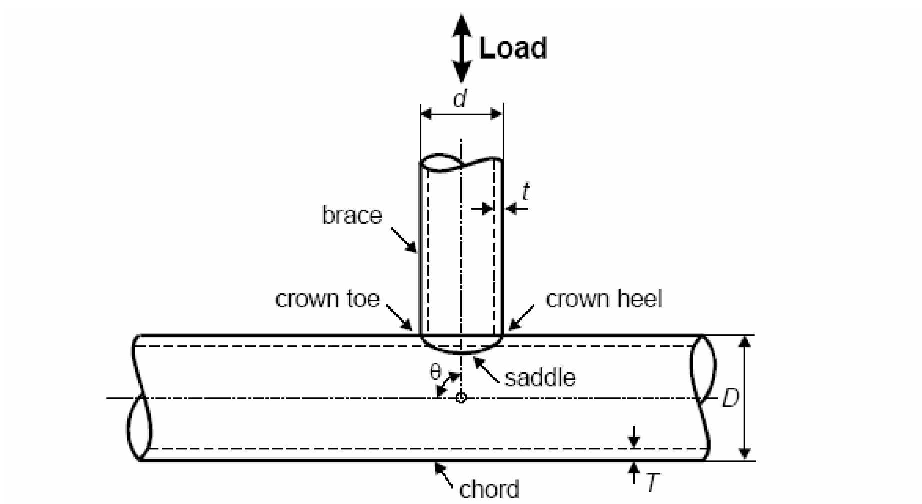
Applicable clause(s):

p. AIII. 21-22

Description: T- and Y-Joints

Loading: Axial

Schematic:



Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$P_c = Q_u Q_f \frac{R_e T^2 K_a}{\sin\theta} \tag{B.148}$$

where

P_C = characteristic strength for brace axial load

R_e = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

$$K_a = \frac{\left(1 + \dfrac{1}{\sin\theta}\right)}{2} \tag{B.149}$$

Q_f = is a factor to allow for the presence of axial and moment loads in the chord. Q_f is defined as:

Q_f = 1.0 - 1.638 $\lambda_\gamma U_2$ for extreme conditions

Q_f = 1.0 - 2.890 $\lambda_\gamma U_2$ for operating conditions

where

λ = 0.030 for brace axial load

λ = 0.045 for brace in-plane moment load

λ = 0.021 for brace out-of-plane moment load

and

$$U = \frac{\sqrt{(0.23 P_a D)^2 + M_{ai}^2 + M_{ao}^2}}{0.72 D^2 T R_e} \tag{B.150}$$

with all forces (P_a, M_{ai}, M_{a0}) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$\text{chord axial tension force} \geq \frac{1}{0.23\text{D}}\left(M_{ai}^2 + M_{ao}^2\right)^{0.5}$ with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$$Q_u = (2 + 20\beta)\sqrt{Q_\beta} \quad \text{(for Axial Compression)} \tag{B.151}$$

$$Q_u = (8 + 22\beta) \quad \text{(for Axial Tension)}$$

Q_β = is the geometric modifier defined as follows

$$Q_\beta = 1.0 \quad \text{for } \beta \leq 0.6$$

$$Q_\beta = \frac{0.3}{\beta(1 - 0.833\beta)} \quad \text{for } \beta > 0.6$$

Bibliography:

[B.39] Offshore Installations: Guidance on Design, Construction and Certification, Fourth Edition, UK Health & Safety Executive, London (1990).

B.8.2 T- and Y-Joints with in-plane and out-of-plane bending

SINTAP

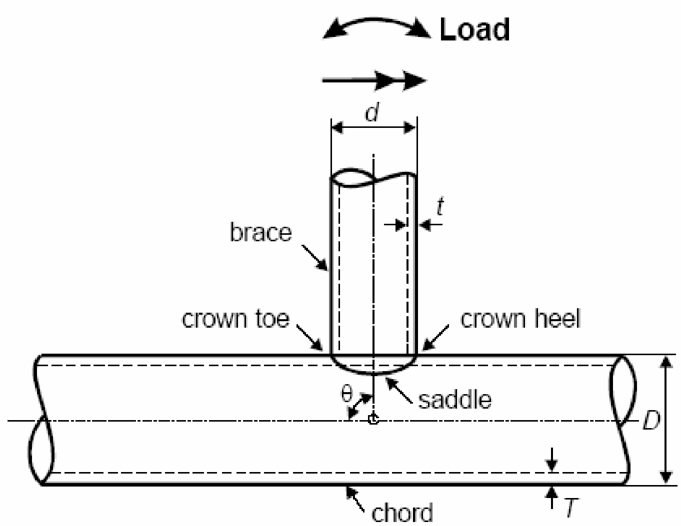
Applicable clause(s):

p. AIII. 23-24

Description: T- and Y-Joints

Loading: In-plane and out-of-plane bending

Schematic:



Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$M_{ci} = M_{co} = Q_u Q_f \frac{R_e T^2 d}{\sin\theta} \tag{B.152}$$

where

M_{ci} = characteristic strength for brace in-plane moment load

M_{co} = characteristic strength for brace out-of-plane moment load

R_e = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

Q_f = is a factor to allow for the presence of axial and moment loads in the chord.

Q_f is defined as:

Q_f = 1.0 - 1.638 $\lambda_\gamma U_2$ for extreme conditions

Q_f = 1.0 - 2.890 $\lambda_\gamma U_2$ for operating conditions

where

λ = 0.030 for brace axial load

λ = 0.045 for brace in-plane moment load

λ = 0.021 for brace out-of-plane moment load

and

$$U = \frac{\sqrt{(0.23P_aD)^2 + M_{ai}^2 + M_{ao}^2}}{0.72D^2TR_e} \qquad \text{(B.153)}$$

with all forces (P_a, M_{ai}, Mao) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$\text{chord axial tension force} \geq \frac{1}{0.23\text{D}}\left(M_{ai}^2 + M_{ao}^2\right)^{0.5}$ with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$Q_u = 5\beta\gamma^{0.5}\sin\theta$ **(for In-Plane Bending)**

$Q_u = (1.6 + 7\beta)Q_\beta$ **(for Out-of Plane Bending)** (B.154)

Q_β = is the geometric modifier defined as follows

$Q_\beta = 1.0$ for β≤0.6

$Q_\beta = \frac{0.3}{\beta(1 - 0.833\beta)}$ for β>0.6

Bibliography:

[B.40] Offshore Installations: Guidance on Design, Construction and Certification, Fourth Edition, UK Health & Safety Executive, London (1990).

B.8.3 K-Joints with axial loads

SINTAP

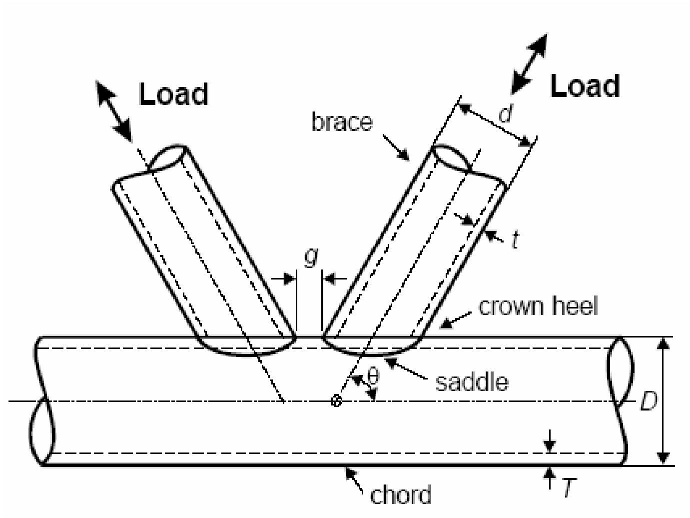
Applicable clause(s):

p. AIII. 25-26

Description: K-Joints

Loading: Axial

Schematic:



Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$P_c = Q_u Q_f \frac{R_e T^2 K_a}{\sin\theta} \tag{B.155}$$

where

P_c = characteristic strength for brace axial load

R_e = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

$$K_a = \frac{\left(1 + \frac{1}{\sin\theta}\right)}{2} \tag{B.156}$$

Q_f = is a factor to allow for the presence of axial and moment loads in the chord. Q_f is defined as:

Q_f = 1.0 - 1.638 $\lambda_\gamma U_2$ for extreme conditions

Q_f = 1.0 - 2.890 $\lambda_\gamma U_2$ for operating conditions

where

λ = 0.030 for brace axial load

λ = 0.045 for brace in-plane moment load

λ = 0.021 for brace out-of-plane moment load

and

$$U = \frac{\sqrt{(0.23 P_a D)^2 + M_{ai}^2 + M_{ao}^2}}{0.72 D^2 T R_e} \quad \text{(B.157)}$$

with all forces (P_a, M_{ai}, *Mao*) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$\text{chord axial tension force} \geq \frac{1}{0.23\text{D}}\left(M_{ai}^2 + M_{ao}^2\right)^{0.5}$ with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$$Q_u = (2 + 20\beta) Q_g \sqrt{Q_\beta} \qquad \textbf{(for Axial Compression)}$$

$$Q_u = (8 + 22\beta) Q_g \qquad \textbf{(for Axial Tension)} \quad \text{(B.158)}$$

Q_β = is the geometric modifier defined as follows

$Q_\beta = 1.0$ for $\beta \leq 0.6$

$Q_\beta = \frac{0.3}{\beta(1 - 0.833\beta)}$ for $\beta > 0.6$

$Q_g = 1.7 - 0.9\zeta^{0.5}$ but should not be taken as less than 1.0.

Bibliography:

[B.41] Offshore Installations: Guidance on Design, Construction and Certification, Fourth Edition, UK Health & Safety Executive, London (1990).

B.8.4 K-Joints with in-plane and out-of-plane bending

SINTAP

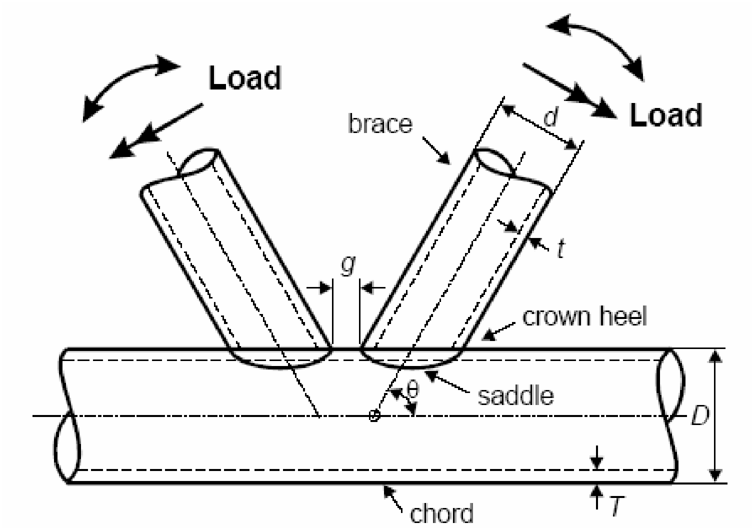
Applicable clause(s):

p. AIII. 27-28

Description: K-Joints

Loading: In-plane and out-of-plane bending

Schematic:



Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$M_{ci} = M_{co} = Q_u Q_f \frac{R_e T^2 d}{\sin\theta} \qquad \text{(B.159)}$$

where

M_{ci} = characteristic strength for brace in-plane moment load

M_{co} = characteristic strength for brace out-of-plane moment load

R_e = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

Q_f = is a factor to allow for the presence of axial and moment loads in the chord.

Q_f is defined as:

Q_f = 1.0 - 1.638 $\lambda_\gamma U_2$ for extreme conditions

Q_f = 1.0 - 2.890 $\lambda_\gamma U_2$ for operating conditions

where

λ = 0.030 for brace axial load

λ = 0.045 for brace in-plane moment load

λ = 0.021 for brace out-of-plane moment load

and

$$U = \frac{\sqrt{(0.23P_a D)^2 + M_{ai}^2 + M_{ao}^2}}{0.72D^2TR_e} \quad \text{(B.160)}$$

with all forces (P_a, M_{ai}, Mao) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$\text{chord axial tension force} \geq \frac{1}{0.23\text{D}}\left(M_{ai}^2 + M_{ao}^2\right)^{0.5}$ with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$Q_u = 5\beta\gamma^{0.5}\sin\theta$ **(for In-Plane Bending)**

$Q_u = (1.6 + 7\beta)Q_\beta$ **(for Out-of Plane Bending)** (B.161)

Q_{β}. = is the geometric modifier defined as follows

$Q_\beta = 1.0$ for β≤0.6

$Q_\beta = \frac{0.3}{\beta(1 - 0.833\beta)}$ for β>0.6

Bibliography:

[B.42] Offshore Installations: Guidance on Design, Construction and Certification, Fourth Edition, UK Health & Safety Executive, London (1990).

B.8.5 X- and DT-Joints with axial load

SINTAP

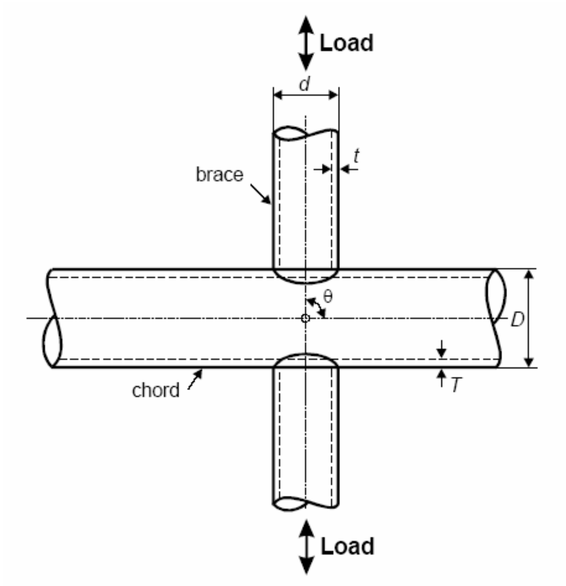
Applicable clause(s):

p. AIII. 29-30

Description: X- and DT-Joints

Loading: Axial

Schematic:



Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$P_c = Q_u Q_f \frac{R_e T^2 K_a}{\sin\theta} \tag{B.162}$$

where

P_c = characteristic strength for brace axial load

R_e = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

$$K_a = \frac{\left(1 + \frac{1}{\sin\theta}\right)}{2} \tag{B.163}$$

Q_f = is a factor to allow for the presence of axial and moment loads in the chord. Q_f is defined as:

Q_f = 1.0 - 1.638 $\lambda_\gamma U_2$ for extreme conditions

Q_f = 1.0 - 2.890 $\lambda_\gamma U_2$ for operating conditions

where

λ = 0.030 for brace axial load

λ = 0.045 for brace in-plane moment load

λ = 0.021 for brace out-of-plane moment load

and

$$U = \frac{\sqrt{(0.23P_a D)^2 + M_{ai}^2 + M_{ao}^2}}{0.72D^2TR_e} \tag{B.164}$$

with all forces (P_a, M_{ai}, *Mao*) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$\text{chord axial tension force} \geq \frac{1}{0.23\text{D}}\left(M_{ai}^2 + M_{ao}^2\right)^{0.5}$ with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$Q_u = (2.5 + 14\beta)Q_\beta$ **(for Axial Compression)**

$Q_u = (7 + 17\beta)Q_\beta$ **(for Axial Tension)** (B.165)

Q_β. = is the geometric modifier defined as follows

$Q_\beta = 1.0$ for β≤0.6

$Q_\beta = \frac{0.3}{\beta(1 - 0.833\beta)}$ for β>0.6

Bibliography:

[B.43] Offshore Installations: Guidance on Design, Construction and Certification, Fourth Edition, UK Health & Safety Executive, London (1990).

B.8.6 X- and DT-Joints with in-plane and out-of-plane bending

SINTAP

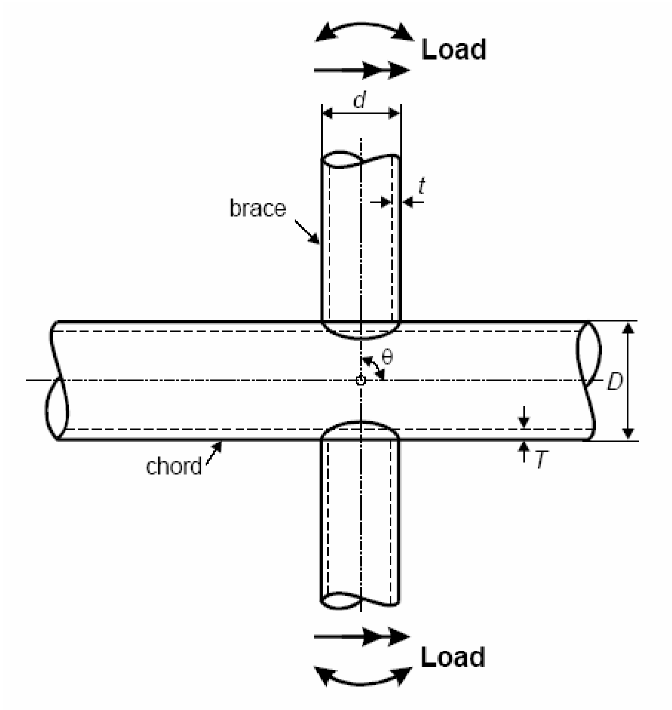
Applicable clause(s):

p. AIII. 31-32

Description: X- and DT-Joints

Loading: In-plane and out-of-plane bending

Schematic:



Limit load Solution:

The characteristic strength of a welded tubular joint subjected to unidirectional loading may be derived as follows:

$$M_{ci} = M_{co} = Q_u Q_f \frac{R_e T^2 d}{\sin\theta} \quad \text{(B.166)}$$

where

M_{ci} = characteristic strength for brace in-plane moment load

M_{co} = characteristic strength for brace out-of-plane moment load

R_e = characteristic yield stress of the chord member at the joint (or 0.7 times the characteristic tensile strength if less). If characteristic values are not available specified minimum values may be substituted.

Q_f = is a factor to allow for the presence of axial and moment loads in the chord.

Q_f is defined as:

Q_f = 1.0 - 1.638 $\lambda_\gamma U_2$ for extreme conditions

Q_f = 1.0 - 2.890 $\lambda_\gamma U_2$ for operating conditions

where

λ = 0.030 for brace axial load

λ = 0.045 for brace in-plane moment load

λ = 0.021 for brace out-of-plane moment load

and

$$U = \frac{\sqrt{(0.23P_aD)^2 + M_{ai}^2 + M_{ao}^2}}{0.72D^2TR_e} \quad \text{(B.167)}$$

with all forces (P_a, M_{ai}, Mao) in the function U relating to the calculated applied loads in the chord. Note that U defines the chord utilisation factor.

Q_f = may be set to 1.0 if the following condition is satisfied:

$\text{chord axial tension force} \geq \frac{1}{0.23\text{D}}\left(M_{ai}^2 + M_{ao}^2\right)^{0.5}$ with all forces relating to the calculated applied loads in the chord.

Q_u = is a strength factor which varies with the joint and load type:

$Q_u = 5\beta\gamma^{0.5}\sin\theta$ **(for In-Plane Bending)**

$Q_u = (1.6 + 7\beta)\sqrt{Q_\beta}$ **(for Out-of Plane Bending)** (B.168)

Q_β. = is the geometric modifier defined as follows

$Q_\beta = 1.0$ for β≤0.6

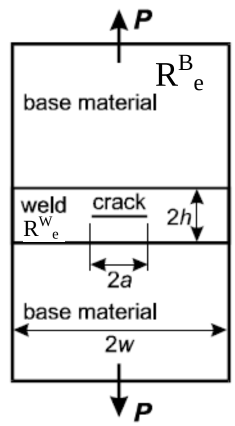
$Q_\beta = \frac{0.3}{\beta(1 - 0.833\beta)}$ for β>0.6

Bibliography:

[B.44] AIII.6. Offshore Installations: Guidance on Design, Construction and Certification,Fourth Edition, UK Health & Safety Executive, London (1990).

B.9 Material mismatch

B.9.1 Crack in the centre line of the weld material



SINTAP

Applicable clause(s):

p. AIV. 4-6

Solution:

(i) Plane Stress

The limit load for the plate made wholly of material *b* is

$$F_e^B = 2R_e^B B(W-a) \tag{B.169}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & \text{for } 0 \le \psi \le 1.43 \\ \min\left\{\dfrac{F_e^{M(1)}}{F_e^B}, \dfrac{F_e^{M(2)}}{F_e^B}\right\} & \text{for } 1.43 \le \psi \end{cases} \tag{B.170}$$

$$\frac{F_e^{M(1)}}{F_e^B} = M\left[\frac{2}{\sqrt{3}} - \left(\frac{2-\sqrt{3}}{\sqrt{3}}\right)\frac{1.43}{\psi}\right] \tag{B.171}$$

$$\frac{F_e^{M(2)}}{F_e^B} = 1-(1-M)\frac{1.43}{\psi} \tag{B.172}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(3)}}{F_e^B}, \frac{1}{1-a/w}\right\} \tag{B.173}$$

$$\frac{F_e^{M(3)}}{F_e^B} = \begin{cases} M & for\ \psi \le \psi_1 = \left(1+0.43e^{-5(M-1)}\right)\cdot e^{-(M-1)/5} \\ \frac{24(M-1)}{25}\frac{\psi_1}{\psi} + \frac{M+24}{25} & for\ \psi \ge \psi_1 = \left(1+0.43e^{-5(M-1)}\right)\cdot e^{-(M-1)/5} \end{cases} \quad \text{(B.174)}$$

(ii) Plane Strain

The limit load for the plate made wholly of material *b* is

$$F_e^B = \frac{4}{\sqrt{3}} R_e^B B(W-a)$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & for\ 0 \le \psi \le 1 \\ \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{F_e^{M(2)}}{F_e^B}\right\} & for\ 1 \le \psi \end{cases} \quad \text{(B.175)}$$

$$\frac{F_e^{M(1)}}{F_e^B} = 1-(1-M)\frac{1}{\psi} \quad \text{(B.176)}$$

$$\frac{F_e^{M(2)}}{F_e^B} = \begin{cases} M\left[1.0+0.462\frac{(\psi-1)^2}{\psi} - 0.04\frac{(\psi-1)^3}{\psi}\right] & for \quad 1 \le \psi \le 3.6 \\ M\left[2.571 - \frac{3.254}{\psi}\right] & for \quad 3.6 \le \psi \le 5.0 \\ M\left[0.125\psi + 1.291 + \frac{0.019}{\psi}\right] & for \quad 5 \le \psi \end{cases} \quad \text{(B.177)}$$

Overmatching (M>1)

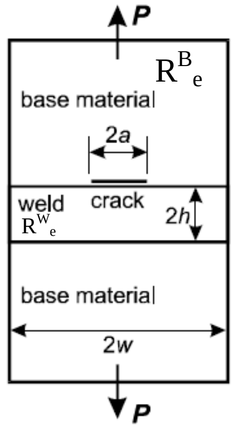
$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(3)}}{F_e^B}, \frac{1}{1-a/w}\right\} \quad \text{(B.178)}$$

$$\frac{F_e^{M(3)}}{F_e^B} = \begin{cases} M & for\ \psi \le \psi_1 = e^{-(M-1)/5} \\ \frac{24(M-1)}{25}\frac{\psi_1}{\psi} + \frac{M+24}{25} & for\ \psi \ge \psi_1 = e^{-(M-1)/5} \end{cases} \quad \text{(B.179)}$$

Bibliography:

[B.45] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.2 Crack in the interface between weld metal and base plate



SINTAP

Applicable clause(s):

p. AIV. 7-8

Solution:

(i) Plane Stress

The limit load for the plate made wholly of material *b* is

$$F_e^B = 2R_e^B B(W-a) \tag{B.180}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = M\left[1.095 - 0.095\exp\left[-(1-M)/0.108M\right]\right] \tag{B.181}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{1}{1-a/w}\right\} \tag{B.182}$$

$$\frac{F_e^{M(1)}}{F_e^B} = 1.095 - 0.095\exp\left[-(1-M)/0.108\right] \tag{B.183}$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$F_e^B = \frac{4}{\sqrt{3}} R_e^B B(W-a) \tag{B.184}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{F_e^{M(2)}}{F_e^B}\right\} \tag{B.185}$$

$$\frac{F_e^{M(1)}}{F_e^B} = \begin{cases} f & for \quad 0 \le \psi \le \psi_1 = 2\left[1-\left(2-\sqrt{2}\right)\left(1-M\right)\right] \\ 1-\left(1-f\right)\dfrac{\psi_1}{\psi} & for\ \psi \ge \psi_1 = 2\left[1-\left(2-\sqrt{2}\right)\left(1-M\right)\right] \end{cases} \tag{B.186}$$

$$f = \begin{cases} M\left[1+0.52\left(\dfrac{1-M}{M}\right)-0.22\left(\dfrac{1-M}{M}\right)^2\right] & for \quad 0.5 \le M \le 1 \\ 1.30M & for \quad M \le 0.5 \end{cases} \tag{B.187}$$

$$\frac{F_e^{M(2)}}{F_e^B} = \begin{cases} 1.30M & for \quad 0 \le \psi \le \sqrt{2} \\ M\left[1.3+0.394\dfrac{\left(\psi-\sqrt{2}\right)^2}{\psi}-0.027\dfrac{\left(\psi-\sqrt{2}\right)^3}{\psi}\right] & for \quad \sqrt{2} \le \psi \le 4.2 \\ M\left[2.881-\dfrac{4.123}{\psi}\right] & for \quad 4.2 \le \psi \le 6.2 \\ M\left[0.125\psi+1.294+\dfrac{0.909}{\psi}\right] & for \quad 6.2 \le \psi \end{cases} \tag{B.188}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(3)}}{F_e^B}, \frac{1}{1-a/w}\right\} \tag{B.189}$$

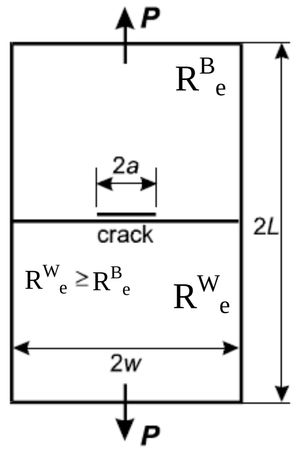
$$\frac{F_e^{M(3)}}{F_e^B} = \begin{cases} f & for \quad 0 \le \psi \le \sqrt{2} \\ \left(f-\dfrac{M+24}{25}\right)\exp\left[-\dfrac{\psi-\sqrt{2}}{4M-1}\right]+\dfrac{M+24}{25} & for\ \psi \ge \sqrt{2} \end{cases} \tag{B.190}$$

$$f = \begin{cases} 1+0.52\left(M-1\right)-0.22\left(M-1\right)^2 & for \quad 1 \le M \le 2 \\ 1.30 & for \quad M \ge 2 \end{cases} \tag{B.191}$$

Bibliography:

[B.46] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.3 Crack in the interface of a bi-material joint



SINTAP

Applicable clause(s):

p. AIV. 9

Solution:

(i) Plane Stress

The limit load for the plate made wholly of material *b* is

$$F_e^B = 2R_e^B B(W-a) \tag{B.192}$$

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{1}{1-a/w}\right\} \tag{B.193}$$

$$\frac{F_e^{M(1)}}{F_e^B} = 1.095 - 0.095\exp\left[-(1-M)/0.108\right] \tag{B.194}$$

(ii) Plane Strain

The limit load for the plate made wholly of material *b* is

$$F_e^B = \frac{4}{\sqrt{3}} R_e^B B(W-a) \tag{B.195}$$

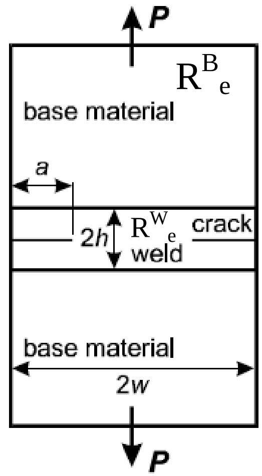
$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{1}{1-a/w}\right\} \tag{B.196}$$

$$\frac{F_e^{M(1)}}{F_e^B} = \begin{cases} 1+0.52(M-1)-0.22(M-1)^2 & for \quad 1 \le M \le 2 \\ 1.30 & for \quad M \ge 2 \end{cases} \tag{B.197}$$

Bibliography:

[B.47] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.4 Crack in the centre line of the weld material



SINTAP

Applicable clause(s):

p. AIV. 10-12

Solution:

(i) Plane Stress

The limit load for the plate made wholly of material b is

$$F_e^B = \beta 2 R_e^B B(W-a);\, \beta = \begin{cases} 1+0.54\left(\frac{a}{w}\right) & for \quad 0 \le \frac{a}{w} \le 0.286 \\ \frac{2}{\sqrt{3}} & for \quad 0.286 \le \frac{a}{w} \le 1 \end{cases} \tag{B.198}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = M \qquad for\, all\, \psi \tag{B.199}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{1}{\beta(1-a/w)}\right\} \tag{B.200}$$

$$\frac{F_e^{M(1)}}{F_e^B} = \begin{cases} M & for \quad 0 \le \psi \le \psi_1 = e^{-2(M-1)/5} \\ \frac{M+24}{25} + \left[\frac{24(M-1)}{25} + 0.1(M-1)\right]\frac{\psi_1}{\psi} - 0.1(M-1)\left(\frac{\psi_1}{\psi}\right)^M & for\ \psi \ge \psi_1 = e^{-2(M-1)/5} \end{cases} \tag{B.201}$$

(ii) Plane Strain

The limit load for the plate made wholly of material *b* is

$$F_e^B = \beta \frac{4}{\sqrt{3}} R_e^B B(W-a); \qquad \beta = \begin{cases} 1+\ln\left(\dfrac{2w-a}{2(w-a)}\right) & \text{for} \quad 0 \le \dfrac{a}{w} \le 0.884 \\ 1+\dfrac{\pi}{2} \quad \text{for} & 0.884 \le \dfrac{a}{w} \le 1 \end{cases} \tag{B.202}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & \text{for } 0 \le \psi \le 0.5 \\ \min\left\{\dfrac{F_e^{M(1)}}{F_e^B}, \dfrac{F_e^{M(2)}}{F_e^B}\right\} & \text{for } 0.5 \le \psi \end{cases} \tag{B.203}$$

$$\frac{F_e^{M(1)}}{F_e^B} = 1-(1-M)\frac{0.5}{\psi} \tag{B.204}$$

$$\frac{F_e^{M(2)}}{F_e^B} = \begin{cases} M\left[\beta + A(\psi-0.5)+B(\psi-0.5)^2\right]/\beta & \text{for} \quad 0.5 \le \psi \le \psi_0 \\ M(0.25\psi+2.2172)/\beta & \text{for} \quad \psi \ge \psi_0 \end{cases} \tag{B.205}$$

$$A = \begin{cases} 0.25 - \dfrac{\beta-2.3422}{\psi_0-0.5} & \text{for} \quad 0 < \dfrac{a}{w} < 0.35 \\ 0.25 - \dfrac{2(\beta-2.3422)}{\psi_0-0.5} & \text{for} \quad 0.35 < \dfrac{a}{w} \end{cases} \tag{B.206}$$

$$B = \begin{cases} 0 & \text{for} \quad 0 < \dfrac{a}{w} < 0.35 \\ \dfrac{\beta-2.3422}{(\psi_0-0.5)^2} & \text{for} \quad 0.35 < \dfrac{a}{w} \end{cases} \tag{B.207}$$

$$\psi_0 = 16.3 - 35.2(a/w) + 19.9(a/w)^2$$

Overmatching (M>1)

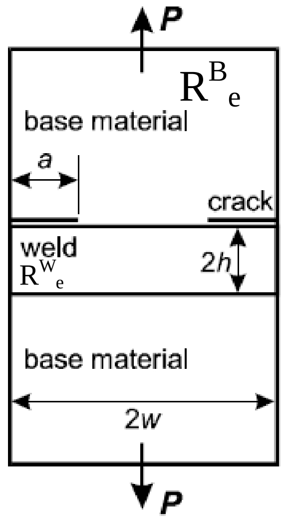
$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(3)}}{F_e^B}, \frac{1}{\beta(1-a/w)}\right\} \tag{B.208}$$

$$\frac{F_e^{M(3)}}{F_e^B} = \begin{cases} M & \text{for} \quad \psi \le \psi_1 = 0.3e^{-(M-1)/5}+0.2 \\ \dfrac{49(M-1)}{50}\dfrac{\psi_1}{\psi} + \dfrac{M+49}{50} & \text{for } \psi \ge \psi_1 = 0.3e^{-(M-1)/5}+0.2 \end{cases} \tag{B.209}$$

Bibliography:

[B.48] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.5 Crack in the interface between weld metal and base plate



SINTAP

Applicable clause(s):

p. AIV. 13-14

Solution:

(i) Plane Stress

The limit load for the plate made wholly of material *b* is

$$F_e^B = \beta 2R_e^B B(W-a);\ \beta = \begin{cases} 1+0.54\left(\dfrac{a}{w}\right) & for \quad 0<\dfrac{a}{w}<0.286 \\ \dfrac{2}{\sqrt{3}} & for \quad 0.286<\dfrac{a}{w}<1 \end{cases} \qquad \text{(B.210)}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = M \qquad for\ all\ \psi \qquad \text{(B.211)}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = 1 \qquad for\ all\ \psi \qquad \text{(B.212)}$$

(ii) Plane Strain

The limit load for the plate made wholly of material *b* is

$$F_e^B = \beta \frac{4}{\sqrt{3}} R_e^B B(W-a); \qquad \beta = \begin{cases} 1+\ln\left(\frac{2w-a}{2(w-a)}\right) & for \quad 0<\frac{a}{w}\le 0.884 \\ 1+\frac{\pi}{2} & for \quad 0.884<\frac{a}{w}<1 \end{cases} \tag{B.213}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & for\ 0\le\psi\le 1 \\ \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{F_e^{M(2)}}{F_e^B}\right\} & for\ 1\le\psi \end{cases} \tag{B.214}$$

$$\frac{F_e^{M(1)}}{F_e^B} = 1-(1-M)\frac{1}{\psi} \tag{B.215}$$

$$\frac{F_e^{M(2)}}{F_e^B} = \begin{cases} M\left[\beta + A(\psi-1) + B(\psi-1)^2\right]/\beta & for \quad 1\le\psi\le\psi_0 \\ M(0.125\psi + 2.2172)/\beta & for \quad \psi\ge\psi_0 \end{cases} \tag{B.216}$$

$$A = \begin{cases} 0.125 - \frac{\beta - 2.3422}{\psi_0 - 1} & for \quad 0<\frac{a}{w}<0.35 \\ 0.125 - \frac{2(\beta-2.3422)}{\psi_0 - 1} & for \quad 0.35<\frac{a}{w} \end{cases} \tag{B.217}$$

$$B = \begin{cases} 0 & for \quad 0<\frac{a}{w}<0.35 \\ \frac{\beta - 2.3422}{(\psi_0 - 1)^2} & for \quad 0.35<\frac{a}{w} \end{cases} \tag{B.218}$$

$$\psi_0 = 32.6 - 70.4(a/w) + 39.8(a/w)^2$$

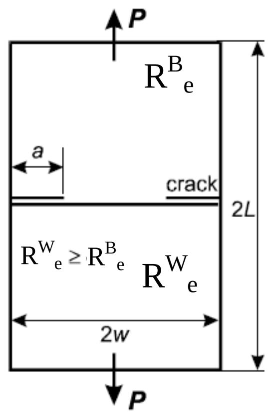
Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = 1 \qquad for\ all\ \psi \tag{B.219}$$

Bibliography:

[B.49] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.6 Crack in the interface of a bi-material joint



SINTAP

Applicable clause(s):

p. AIV. 15

Solution:

(i) Plane Stress

$$F_e^B = \beta 2 R_e^B B(W-a); \qquad \beta = \begin{cases} 1+0.54\left(\frac{a}{w}\right) & for \quad 0<\frac{a}{w}<0.286 \\ \frac{2}{\sqrt{3}} & for \quad 0.286<\frac{a}{w}<1 \end{cases} \tag{B.220}$$

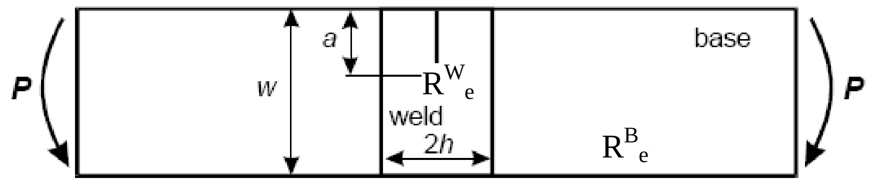
(ii) Plane Strain

$$F_e^M = \beta \frac{4}{\sqrt{3}} R_e^B B(W-a); \qquad \beta = \begin{cases} 1+\ln\left(\frac{2w-a}{2(w-a)}\right) & for \quad 0<\frac{a}{w}<0.884 \\ 1+\frac{\pi}{2} & for \quad 0.884<\frac{a}{w}<1 \end{cases} \tag{B.221}$$

Bibliography:

[B.50] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.7 Crack in the centre line of the weld material



Crack in the centre line of the weld material

SINTAP

Applicable clause(s):

p. AIV. 16-18

Solution:

(i) Plane Stress

The limit load for the plate made wholly of material *b* is

$$F_e^B = 0.4641 \frac{R_e^B}{\sqrt{3}} B(W-a)^2 \tag{B.222}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = M \qquad for\, all\, \psi \tag{B.223}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{1}{(1-a/w)^2}\right\} \tag{B.224}$$

$$\frac{F_e^{M(1)}}{F_e^B} = \begin{cases} M & for \quad 0 \le \psi \le \psi_1 \\ \dfrac{M+49}{50} + \left(\dfrac{49(M-1)}{50} + 1 - \sqrt{M-1}\right)\dfrac{\psi_1}{\psi} + \left(1+\sqrt{M-1}\right)\left(\dfrac{\psi_1}{\psi}\right)^M & for\ \psi \ge \psi_1 \end{cases}$$

$$\psi_1 = \left(2.0 + 0.7e^{-(M-1)}\right)e^{-(M-1)/8} \tag{B.225}$$

(ii) Plane Strain

The limit load for the plate made wholly of material *b* is

$$F_e^B = \beta \frac{R_e^B}{\sqrt{3}} B(W-a)^2 ; \beta = \begin{cases} 0.5 + 0.808\left(\frac{a}{w}\right) - 1.245\left(\frac{a}{w}\right)^2 & for \quad 0 < \frac{a}{w} < 0.3 \\ 0.631 \quad for & 0.3 < \frac{a}{w} < 1 \end{cases} \tag{B.226}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & for\ 0 \le \psi \le 2.0 \\ \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{F_e^{M(2)}}{F_e^B}\right\} & for\ 2.0 \le \psi \end{cases} \tag{B.227}$$

$$\frac{F_e^{M(1)}}{F_e^B} = \left[\frac{9(M-1)}{10}\right]\exp\left[-\frac{1}{20(1-M)}(\psi - 2)\right] + \frac{M+9}{10} \tag{B.228}$$

For 0<a/w≤0.3

$$\frac{F_e^{M(2)}}{F_e^B} = \begin{cases} M\left[1 + \frac{-3\beta + 5.4}{1.69\beta}\left(\frac{\psi}{10} - 0.2\right)^2 - \frac{2\beta + 3.33}{2.2\beta}\left(\frac{\psi}{10} - 0.2\right)^3\right] & for \quad 2.0 \le \psi \le 15.0 \\ M\left(1.1345 + 0.623\frac{\psi}{10}\right)\Big/\beta & for \quad \psi \ge 15 \end{cases} \tag{B.229}$$

For 0.3<a/w

$$\frac{F_e^{M(2)}}{F_e^B} = \begin{cases} M\left[1.094 - 1.017\left(\frac{\psi}{10}\right) + 3.129\left(\frac{\psi}{10}\right)^2 - 1.952\left(\frac{\psi}{10}\right)^3\right] & for \quad 2.0 \le \psi \le 7.0 \\ M\left(0.900 + 0.494\frac{\psi}{10}\right) & for \quad \psi \ge 7.0 \end{cases} \tag{B.230}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(3)}}{F_e^B}, \frac{1}{2\beta(1-a/w)^2}\right\} \tag{B.231}$$

$$\frac{F_e^{M(3)}}{F_e^B} = \begin{cases} M & for \quad 0 \le \psi \le \psi_1 \\ A + B\frac{\psi_1}{\psi} + C\left(\frac{\psi_1}{\psi}\right)^M & for\ \psi \ge \psi_1 \end{cases} \tag{B.232}$$

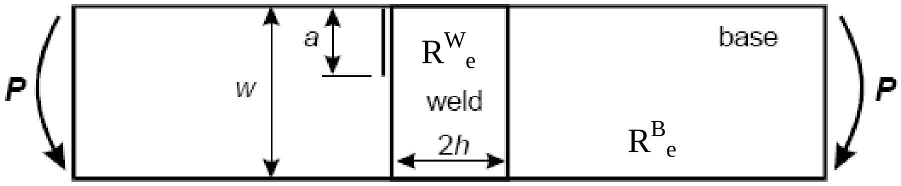
$$\psi_1 = \begin{cases} 2e^{-(M-1)/(10a/w)} & for \quad 0 < a/w \le 0.4 \\ 2e^{-(M-1)/8} & for \quad 0.4 < a/w \end{cases} \tag{B.233}$$

$$A = \frac{M+49}{50} ; B = \frac{49(M-1)}{50} - C ; C = 0.3(M-1)\sqrt{M-1}$$

Bibliography:

[B.51] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.8 Crack in the interface between weld metal and base plate



Crack in the interface between weld metal and base plate

SINTAP

Applicable clause(s):

p. AIV. 19-20

Solution:

(i) Plane Stress

The limit load for the plate made wholly of material *b* is

$$F_e^B = 0.4641\frac{R_e^B}{\sqrt{3}}B(W-a)^2 \tag{B.234}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = M\left[1.04 - 0.04e^{-(1-M)/0.13M}\right] \qquad for\ all\ \psi \tag{B.235}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = 1.04 - 0.04e^{-(1-M)/0.13M} \qquad for\ all\ \psi \tag{B.236}$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$F_e^B = \beta\frac{R_e^B}{\sqrt{3}}B(W-a)^2;\ \beta = \begin{cases} 0.5 + 0.808\left(\dfrac{a}{w}\right) - 1.245\left(\dfrac{a}{w}\right)^2 & for \quad 0 < \dfrac{a}{w} < 0.3 \\ 0.631 \quad for & 0.3 \le \dfrac{a}{w} < 1 \end{cases} \tag{B.237}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & for\ 0 \le \psi \le 4 \\ \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{F_e^{M(2)}}{F_e^B}\right\} & for\ 4 \le \psi \end{cases} \quad \text{(B.238)}$$

$$\frac{F_e^{M(1)}}{F_e^B} = Ae^{-B(\psi-4)} + C$$

$$f = \begin{cases} M & for \quad 0 < a/w \le 0.3 \\ M \cdot \left[1.06 - 0.06e^{-(1-M)/0.3M}\right] & for \quad 0.3 \le a/w \end{cases} \quad \text{(B.239)}$$

$$A = (f - C)[1 + B(\psi - 4)];\ B = \frac{1}{8.5\sqrt{1-M}};\ C = \frac{M+9}{10} \quad \text{(B.240)}$$

For 0<a/w≤0.3

$$\frac{F_e^{M(2)}}{F_e^B} = \begin{cases} M\left[1 + \frac{2\beta - 3.377}{\beta}\left(\frac{\psi}{10}\right)^2 + \frac{-3\beta + 5.377}{\beta}\left(\frac{\psi}{10}\right)^3\right] & for \quad 4.0 \le \psi \le 14.0 \\ M\left(1.377 + 0.623\frac{\psi}{10}\right)\Big/\beta & for \quad \psi \ge 14 \end{cases} \quad \text{(B.241)}$$

For 0.3≤a/w

$$\frac{F_e^{M(2)}}{F_e^B} = \begin{cases} M\left[1.06 + 0.522\left(\frac{\psi}{10}\right)^2 - 0.133\left(\frac{\psi}{10}\right)^3\right] & for \quad 4.0 \le \psi \le 14.0 \\ M\left(1 + 0.494\frac{\psi}{10}\right) & for \quad \psi \ge 14.0 \end{cases} \quad \text{(B.242)}$$

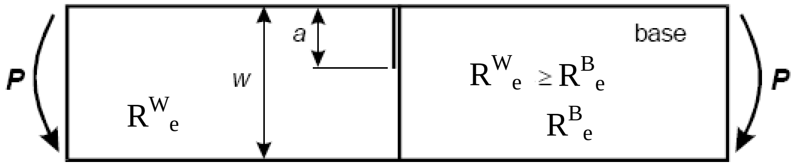
Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} \approx \begin{cases} 1 & for\ 0 < \frac{a}{w} < 0.3 \\ -0.06e^{-(M-1)/0.3} + 1.06 & for \quad 0.3 \le \frac{a}{w} \end{cases} \quad \text{(B.243)}$$

Bibliography:

[B.52] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.9 Crack in the interface of a bi-material joint



Crack in the interface of a bimaterial joint

SINTAP

Applicable clause(s):

p. AIV. 21

Solution:

(i) Plane Stress

$$F_e^M = 0.4641\beta \frac{R_e^B}{\sqrt{3}} B(W-a)^2 \;;\; \beta = 1.04 - 0.04e^{-(M-1)/0.13} \tag{B.244}$$

(ii) Plane Strain

$$F_e^B = \beta \frac{R_e^B}{\sqrt{3}} B(W-a)^2 ; \beta = \begin{cases} (\beta_1 - \beta_\infty)e^{-(M-1)/(a/w)} + \beta_\infty & for \quad 0 < \frac{a}{w} \leq 0.3 \\ (\beta_1 - \beta_\infty)e^{-(M-1)/0.3} + \beta_\infty & for \quad 0.3 < \frac{a}{w} \leq 1 \end{cases} \tag{B.245}$$

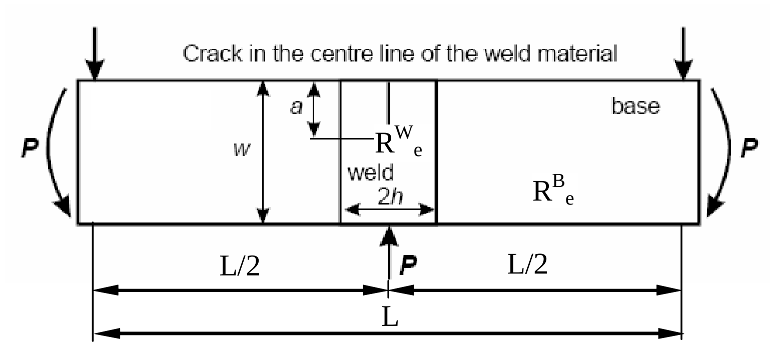
$$\beta_1 = \begin{cases} 0.500 + 0.808\left(\frac{a}{w}\right) - 1.245\left(\frac{a}{w}\right)^2 & for \quad 0 < \frac{a}{w} \leq 0.3 \\ 0.631 \quad for \quad 0.3 < \frac{a}{w} \leq 1 \end{cases} \tag{B.246}$$

$$\beta_\infty = \begin{cases} 0.500 + 0.890\left(\frac{a}{w}\right) - 1.165\left(\frac{a}{w}\right)^2 & for \quad 0 < \frac{a}{w} \leq 0.4 \\ 0.670 \quad for \quad 0.4 < \frac{a}{w} \leq 1 \end{cases} \tag{B.247}$$

Bibliography:

[B.53] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.10 Crack in the centre line of the weld metal



SINTAP

Applicable clause(s):

p. AIV. 22-24

Solution:

(i) Plane Stress

The limit load for the plate made wholly of material *b* is

$$F_e^B = 0.960 \frac{R_e^B}{\sqrt{3}} \frac{B(W-a)^2}{L/2} \tag{B.248}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = M \qquad for\ all\ \psi \tag{B.249}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{F_e^{M(2)}}{F_e^B}\right\} \tag{B.250}$$

$$\frac{F_e^{M(1)}}{F_e^B} = \begin{cases} M & for \quad 0 \le \psi \le \psi_1 \\ \dfrac{M+49}{50} + \left(\dfrac{49(M-1)}{50} - 0.2\sqrt{M-1}\right)\dfrac{\psi_1}{\psi} + 0.2(M-1)\left(\dfrac{\psi_1}{\psi}\right)^M & for\ \psi \ge \psi_1 \end{cases}$$

$$\psi_1 = \left(2.5 + 0.5e^{-(M-1)}\right)e^{-(M-1)/4} \tag{B.251}$$

$$\frac{F_e^{M(2)}}{F_e^B} = \frac{\beta_b}{0.960} \frac{1}{(1-a/w)^2}\ ;\ \beta_b = 4.00 - 2.60\left(2 - \frac{H}{W}\right) + 0.54\left(2 - \frac{H}{W}\right)^2 \tag{B.252}$$

(ii) Plane Strain

The limit load for the plate made wholly of material b is

$$F_e^B = \beta \frac{R_e^B}{\sqrt{3}} \frac{B(W-a)^2}{L/2}; \beta = \begin{cases} 1.125 + 0.892\left(\frac{a}{w}\right) - 2.238\left(\frac{a}{w}\right)^2 & for \quad 0 < \frac{a}{w} < 0.172 \\ 1.199 + 0.096\left(\frac{a}{w}\right) & for \quad 0.172 \le \frac{a}{w} < 1 \end{cases} \tag{B.253}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & for\ 0 < \psi < 2.0 \\ \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{F_e^{M(2)}}{F_e^B}\right\} & for\ 2.0 \le \psi \end{cases} \tag{B.254}$$

$$\frac{F_e^{M(1)}}{F_e^B} = \begin{cases} M\left[1 + \frac{-3\beta + 5.384}{\beta}\left(\frac{\psi}{10} - 0.2\right)^2 + \frac{2\beta - 3.384}{\beta}\left(\frac{\psi}{10} - 0.2\right)^3\right] & for \quad 2.0 \le \psi \le 12.0 \\ M\left(1.384 + 0.616\left(\frac{\psi}{10} - 0.2\right)\right)\Big/\beta & for \quad \psi \ge 12 \end{cases} \tag{B.255}$$

$$\frac{F_e^{M(2)}}{F_e^B} = \frac{9(M-1)}{10}\exp\left[-\frac{1}{20(1-M)}(\psi - 2)\right] + \frac{M+9}{10} \tag{B.256}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(3)}}{F_e^B}, \frac{F_e^{M(4)}}{F_e^B}\right\} \tag{B.257}$$

$$\frac{F_e^{M(3)}}{F_e^B} = \frac{M+49}{50} + \left(\frac{49(M-1)}{50} - 0.3(M-1)\sqrt{M-1}\right)\frac{\psi_1}{\psi} + 0.3(M-1)\sqrt{M-1}\left(\frac{\psi_1}{\psi}\right)^M \tag{B.258}$$

$$\psi_1 = \begin{cases} 2e^{-(M-1)/(4a/w)} & for \quad 0 < a/w < 0.172 \\ 2e^{-(M-1)/8} & for \quad 0.172 \le a/w < 1 \end{cases} \tag{B.259}$$

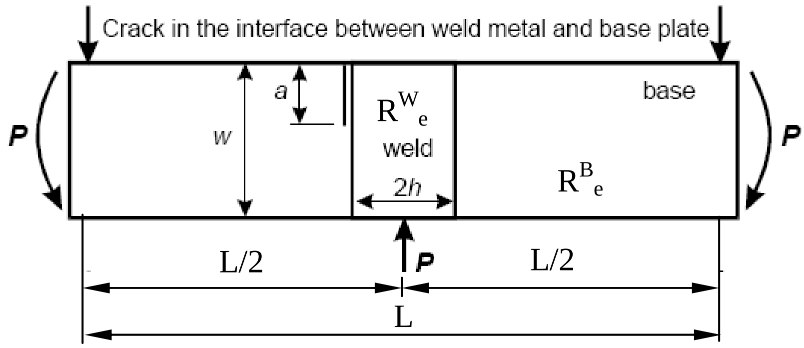
$$\frac{F_e^{M(4)}}{F_e^B} = \frac{\beta_b}{\beta}\frac{1}{(1-a/w)^2}; \tag{B.260}$$

$$\beta_b = 4.5557 - 3.6072\left(2 - \frac{H}{W}\right) + 1.3095\left(2 - \frac{H}{W}\right)^2 - 0.1818\left(2 - \frac{H}{W}\right)^3 \tag{B.261}$$

Bibliography:

[B.54] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.11 Crack in the interface between weld metal and base plate



SINTAP

Applicable clause(s):

p. AIV. 25-26

Solution:

(i) Plane Stress

The limit load for the plate made wholly of material *b* is

$$F_e^B = 0.960 \frac{R_e^B}{\sqrt{3}} \frac{B(W-a)^2}{L/2} \tag{B.262}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = M \qquad for\ all\ \psi \tag{B.263}$$

Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = 1 \qquad for\ all\ \psi \tag{B.264}$$

(ii) Plane Strain

The limit load for the plate made wholly of material *b* is

$$F_e^B = \beta \frac{R_e^B}{\sqrt{3}} \frac{B(W-a)^2}{L/2}; \beta = \begin{cases} 1.125 + 0.892\left(\frac{a}{w}\right) - 2.238\left(\frac{a}{w}\right)^2 & for \quad 0 < \frac{a}{w} < 0.172 \\ 1.199 + 0.096\left(\frac{a}{w}\right) & for \quad 0.172 \le \frac{a}{w} < 1 \end{cases} \tag{B.265}$$

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & \text{for } 0 < \psi < 4.0 \\ \min\left\{\dfrac{F_e^{M(1)}}{F_e^B}, \dfrac{F_e^{M(2)}}{F_e^B}\right\} & \text{for } 4.0 \le \psi \end{cases} \tag{B.266}$$

$$\frac{F_e^{M(1)}}{F_e^B} = \begin{cases} M\left[1 + \dfrac{-3\beta + 9.08}{8\beta}\left(\dfrac{\psi}{10} - 0.4\right)^2 + \dfrac{\beta - 2.616}{16\beta}\left(\dfrac{\psi}{10} - 0.4\right)^3\right] & \text{for} \quad 4.0 \le \psi \le 12.0 \\ M\left(2.0 + 0.616\left(\dfrac{\psi}{10} - 0.4\right)\right)\Big/\beta & \text{for} \quad \psi \ge 12 \end{cases} \tag{B.267}$$

$$\frac{F_e^{M(2)}}{F_e^B} = \frac{9(M-1)}{10}\exp\left[-\frac{1}{20(1-M)}(\psi - 4)\right] + \frac{M+9}{10} \tag{B.268}$$

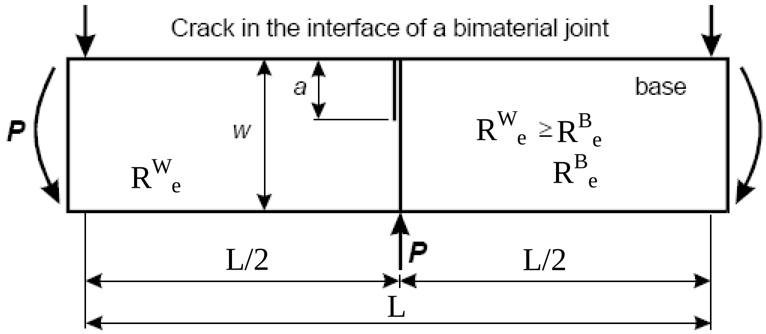
Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = 1 \qquad \text{for all } \psi \tag{B.269}$$

Bibliography:

[B.55] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.12 Crack in the interface of a bi-material joint



SINTAP

Applicable clause(s):

p. AIV. 27

Solution:

(i) Plane Stress

$$F_e^M = 0.960 \frac{R_e^B}{\sqrt{3}} \frac{B(W-a)^2}{L/2} \quad \text{(B.270)}$$

(ii) Plane Strain

$$F_e^M = \beta \frac{R_e^B}{\sqrt{3}} \frac{B(W-a)^2}{L/2} ; \beta = (\beta_1 - \beta_\infty) e^{-(M-1)/0.23} + \beta_\infty \quad \text{(B.271)}$$

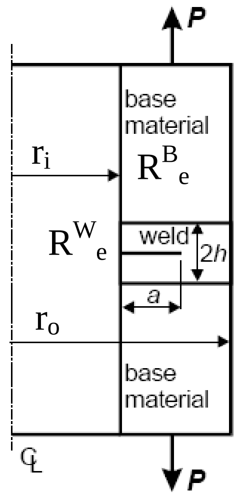
$$\beta_1 = \begin{cases} 1.125 + 0.892\left(\frac{a}{w}\right) - 2.238\left(\frac{a}{w}\right)^2 & for \quad 0 < \frac{a}{w} \le 0.172 \\ 1.199 + 0.096\left(\frac{a}{w}\right) & for \quad 0.172 < \frac{a}{w} \le 1 \end{cases} \quad \text{(B.272)}$$

$$\beta_\infty = \begin{cases} 1.125 + 1.108\left(\frac{a}{w}\right) - 2.072\left(\frac{a}{w}\right)^2 & for \quad 0 < \frac{a}{w} \le 0.172 \\ 1.238 + 1.107\left(\frac{a}{w}\right) & for \quad 0.172 < \frac{a}{w} \le 1 \end{cases} \quad \text{(B.273)}$$

Bibliography:

[B.56] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.13 Crack in the centre line of the weld metal



SINTAP

Applicable clause(s):

p. AIV. 28-29

Solution:

The limit load for the pipe made wholly of material *b* is

$$F_e^B = 2\frac{R_e^B}{\sqrt{3}}\pi\left[r_o^2-(r_i+a)^2\right] \tag{B.274}$$

Undermatching (M<1)

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & for\ 0\le\psi\le 1 \\ \min\left\{\frac{F_e^{M(1)}}{F_e^B},\frac{F_e^{M(2)}}{F_e^B}\right\} & for\ 1\le\psi \end{cases} \tag{B.275}$$

$$\frac{F_e^{M(1)}}{F_e^B} = M\left[1+\frac{\psi-1}{3\sqrt{3}}\right] \tag{B.276}$$

$$\frac{F_e^{M(2)}}{F_e^B} = 1-(1-M)\frac{1}{\psi} \tag{B.277}$$

Overmatching (M>1)

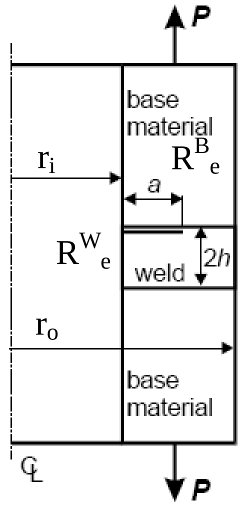
$$\frac{F_e^M}{F_e^B} = \min\left\{\frac{F_e^{M(3)}}{F_e^B},\frac{1}{1-a/w}\right\} \tag{B.278}$$

$$\frac{F_e^{M(3)}}{F_e^B} = \begin{cases} M & for\ \psi \le \psi_1 = e^{-2(M-1)/5} \\ \frac{24(M-1)}{25} \cdot \frac{\psi_1}{\psi} + \frac{M+24}{25} & for\ \psi \ge \psi_1 = e^{-2(M-1)/5} \end{cases} \qquad \text{(B.279)}$$

Bibliography:

[B.57] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.14 Crack in the interface between weld metal and base pipe



SINTAP

Applicable clause(s):

p. AIV. 30

Solution:

The limit load for the pipe made wholly of material *b* is

$$F_e^B = 2\frac{R_e^B}{\sqrt{3}}\pi\left[r_o^2 - (r_i + a)^2\right] \tag{B.280}$$

$$\frac{F_e^M}{F_e^B} = \begin{cases} M & for\ 0 \le \psi \le 2 \\ \min\left\{\frac{F_e^{M(1)}}{F_e^B}, \frac{F_e^{M(2)}}{F_e^B}\right\} & for\ 2 \le \psi \end{cases} \tag{B.281}$$

$$\frac{F_e^{M(1)}}{F_e^B} = M\left[1 + \frac{\psi - 2}{6\sqrt{3}}\right] \tag{B.282}$$

$$\frac{F_e^{M(2)}}{F_e^B} = 1 - (1 - M)\frac{2}{\psi} \tag{B.283}$$

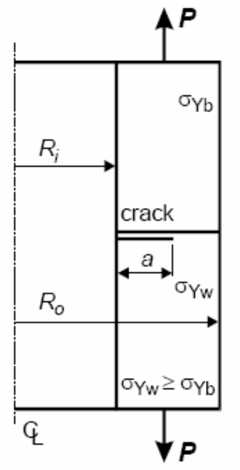
Overmatching (M>1)

$$\frac{F_e^M}{F_e^B} = 1 \qquad for\ all\ \psi \tag{B.284}$$

Bibliography:

[B.58] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.15 Crack in the interface of a bi-material joint



SINTAP

Applicable clause(s):

p. AIV. 31

Solution:

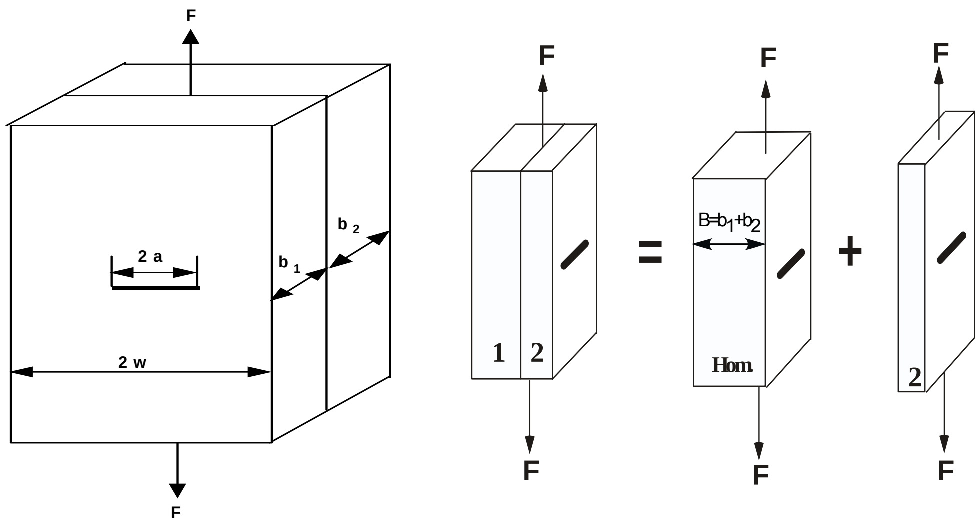
$$F_e^M = 2\frac{R_e^B}{\sqrt{3}}\pi\left[r_o^2 - (r_i + a)^2\right] \tag{B.285}$$

Remarks: Solutions are valid for thin-walled pipes with deep cracks, a/t≥0.3

Bibliography:

[B.59] H. Schwalbe, Y.-J. Kim, S. Hao, and A. Cornec, ETM-MM – The Engineering Treatment Model for Mis-Matched Welded Joints, Mis-Matching of Welds, ESIS 17, Edited by K.-H. Schwalbe and M. Koçak, Mechanical Engineering Publications, London, 539-560 (1994).

B.9.16 Bi-material (clad) center through thickness cracked plate under tension



Applicable clause(s):

For a centre cracked clad plate, Alexandrov and Kocak proposed a closed-form expression for the tensile limit load under plane stress condition.

Solution:

$$F_e^{bi-layer} / F_e^{(h)} = 1/4 b_2 B^{-1} (1 + Mb)(2 + R_e^2 / k_2) \qquad \text{(B.286)}$$

where

$F_e^{bi-layer}$ is the tensile limit load of the whole bi-layer system

$F_e^{(h)} = 2BR_e^2 (W - a)$ is the tensile limit load of the homogeneous centre cracked plate made of the softer material (here material 2) with yield strength R_e^2, thickness of B=b_1+b_2, half width of W (Fig. (11.9.1)) and shear yield strength of k_2

$b = b_1 / b_2$,

$M = R_e^1 / R_e^2$ is strength mis-match ratio with R_e^1 the yield strength of the higher yield strength material (here Material 1).

Alternatively, limit load of the bi-layer plate can be derived using rule of mixtures as AK. Motarjemi, M. Kocak proposed.

By substituting the corresponding values of b, M, k_2, $F_e^{(h)}$ and B into the equation above, and after some simplifications, based on the Tresca yield criteria ($\sigma_2 = 2k_2$), a simple tensile yield load solution, can be found for the bi-layer plate as:

$$\frac{F_e^{bi-layer}}{F_e^{(h)}} = \frac{b_2}{b_1 + b_2}\left(1 + \frac{F_e^1}{F_e^2}\right) \tag{B.287}$$

where

$$F_e^{(h)} = 2R_e^2 (b_1 + b_2)(W - a) \tag{B.288}$$

and after simplification:

$$F_e^{bi-layer} = \sum_{i=1}^{n} F_e^i \tag{B.289}$$

in which:

$F_e^{bi-layer}$ is as defined earlier and F_e^i is the tensile limit load for the centre-cracked configuration of each constituent of the bi-layer system, i.e., F_e^1 and F_e^2.

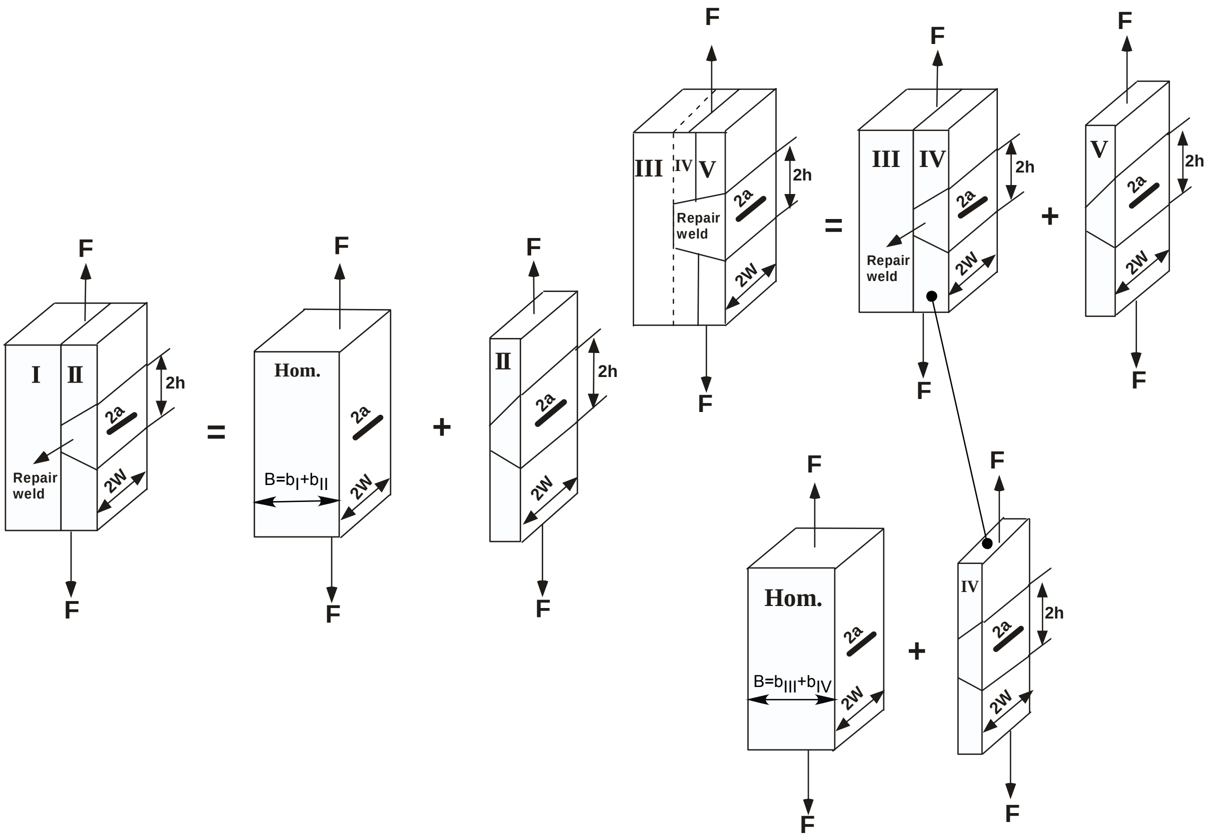
References

[B.60] S. Alexandrov, M. Kocak, “Limit load solutions for bilayer plates with a through crack subject to tension”, Engineering Fracture Mechanics 64 (1999) 507-511.

[B.61] AK. Motarjemi, M. Kocak, “Tensile yield load solutions for centre cracked bilayer (clad) plates with and without repair welds”, Science and Technology of Welding and Joining, 2002, Vol.7, No 5, 299-305

[B.62] AK. Motarjemi and M. Kocak, “Fracture assessment of a clad steel using various SINTAP defect assessment procedure levels”, 2002 Fatigue Fract Engng Mater Struct 25, 929-939

B.9.17 Centre cracked bi-layer (clad) plate with repair weld



Case1 *Case2*

Applicable clause(s):

By using this approach, the limit load of the repair welded bi-layer plates has been derived by Moterjemi and Kocak and SINTAP fracture assessment is verified.

Solution:

For all these cases, it is assumed that the centre cracked bi-layer plate is made of two parts; one without a repair weld (homogeneous), e.g., parts I and III, and the other one with a repair weld (mis-matched), e.g., parts II, IV and V. Based on this assumption, the tensile yield load solution for the whole system, $F_e^{bi-layer}$, can be written as:

$$F_e^{bi-layer} = F_e^{(h)} + F_e^{M} \tag{B.290}$$

Where $F_e^{(h)}$ and F_e^{M} are the tensile yield load values respectively for the homogeneous and mis-matched parts of the bi-layer system.

For the yield load solution for the homogeneous part it can be written:

$$F_e^{(h)} = 2R_e^I (b_I + b_{II})(W - a), \qquad \text{(for the case 1)} \tag{B.291}$$

and

$$F_e^{(h)} = 2R_e^{III}\left(b_{III} + b_{IV}\right)\left(W - a\right), \qquad \text{(for the case 2)} \tag{B.292}$$

where all the parameters have been defined in figure above.

For the mis-matched parts (II, IV and V), tensile yield load solutions for these configurations, the plane stress tensile yield load solution for an over-matching condition, F_e^M , is as follows:

$$\frac{F_e^M}{F_e^B} = Min\{\alpha, \beta\}$$

where

$$\alpha = \begin{cases} M & for \quad \psi \le \psi_1 = \left(1 + 0.43e^{-5(M-1)}\right)e^{-(M-1)/5} \\ \dfrac{24(M-1)}{25} \cdot \dfrac{\psi_1}{\psi} + \dfrac{M+24}{25} & for \quad \psi > \psi_1 = \left(1 + 0.43e^{-5(M-1)}\right)e^{-(M-1)/5} \end{cases} \tag{B.293}$$

$\psi = (W - a)/h$ .with 2h equal to the total width of the weld.

$M = R_e^W / R_e^B$ is the strength mis-match ratio, with R_e^B as the yield strength of the base and R_e^W as the yield strength of the weld materials in the mis-matched (repair welded) parts

$$\beta = \frac{1}{1 - a/W} \tag{B.294}$$

$$F_e^B = 2R_e^B B(W - a) \tag{B.295}$$

References

[B.63] S. Alexandrov, M. Kocak, "Limit load solutions for bilayer plates with a through crack subject to tension", Engineering Fracture Mechanics 64 (1999) 507-511.

[B.64] AK. Motarjemi, M. Kocak, "Tensile yield load solutions for centre cracked bilayer (clad) plates with and without repair welds", Science and Technology of Welding and Joining, 2002, Vol.7, No 5, 299-305

[B.65] AK. Motarjemi and M. Kocak, "Fracture assessment of a clad steel using various SINTAP defect assessment procedure levels", 2002 Fatigue Fract Engng Mater Struct 25, 929-939

Bibliography

[B.66] Ref

[B.67] Ref

[B.68] Ref

[B.69] Ref