Annex H

Probability and Reliability Principles

H.1 Introduction

The application of deterministic fracture mechanics assessment procedures to the prediction of fitness-for-purpose requires the use of data that are often subject to considerable uncertainty. The use of extreme bounding values for the relevant parameters can lead, in some circumstances, to unacceptably over-conservative predictions of structural integrity. An alternative approach is to use reliability methods to allow for the uncertainties in the parameters and to assess the probability of failure of structures containing flaws. It should be noted that the question of the required reliability or safety margin for a particular application depends on the consequences of the failure and requires an overall risk assessment to be carried out.

The procedures described in Sections 6 - 8 are, with the exception of the treatment of fracture toughness data, deterministic. The input data are treated as a set of fixed quantities and the result obtained is unique to these data. Different forms of a result can be obtained but in all cases a comparison of the result with a perceived critical state is performed. Since the perceived critical state is dependent on choice of analysis, and contains an inherent degree of conservatism, it is best regarded as a limiting condition rather than critical.

In this section, methods of probabilistic defect assessment are described which enable the probability of failure to be calculated for a given set of parameters and their corresponding statistical distribution. In addition, a method based on the use of recommended partial safety factors on input parameters for a range of target failure probabilities is presented.

H.2 Applications in Fatigue Assessment

From the very beginning fatigue assessments have considered fatigue resistance variability (dispersion) by the statistical treatment of the fatigue test results and the use of a design S-N curve so that in practice, no sample failure may be expected (see 7.2.1.4). But variability is a characteristic not only of the fatigue resistance. Many other parameters considered in the fatigue assessment present a large variability such as, loads, stress calculations, etc …

The aim of this annex is to provide a review of the reliability principles and of the probabilistic approach which allows all known and possible to model variables used in fatigue assessment to be taken into account and to determine the failure probability during a selected life time span. Both approaches are considered, S-N curve and crack propagation assessment methods.

The following areas are considered:

- safety philosophy
- statistical treatment of data
- probabilistic approach
- semi-probabilistic format
- risk based inspection (RBI)

H.2 Safety philosophy

Many different design philosophies have been developed over the years. Depending on the grade of accuracy in defining the fatigue life approach for specific structures and components several methods are available. The choice of the proper design approach is mostly driven by the risk for human lives and economic reasons: the need is to prevent human life losses and to minimize production and maintenance costs by guaranteeing the required level of safety. The failure criteria to be considered during a design is defined from the acceptable probability level of failure, which implies knowledge of the probability distributions of the various parameters involved and the consequences with respect to economic or human life losses.

H.2.1 Safe life design

This design strategy is based on the assumption that initially the structure is free of imperfections. No regular monitoring in service is specified. When components need to maintain their integrity for a certain amount of cycles during a limited service time designing the components for an infinite life would lead to an extremely heavy and inefficient construction. Therefore, the assessment can be performed by calculating a Miners sum or an equivalent stress range. When using Miner's sum (see 7.3 – route 1, 2 or 3), the allowable value,D_u, is defined as the value which would be expected to cause failure at the desired lifetime, for example:

$D_u = 0.5$ on a 20 years life time

When using the equivalent stress amplitude or range depending of the applied route, (see 7.6.1.3.2), the allowable fatigue stress level is defined, in the same way, as the stress which would be expected to cause the failure at the desired lifetime (see Figure H. 1:).

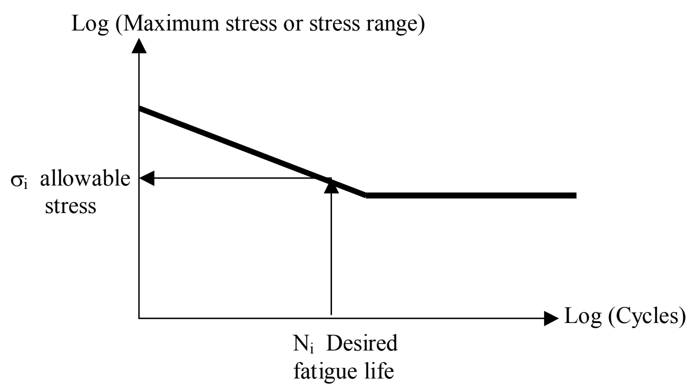

Figure H. 1: The Safe-Life approach

Inherent to the limit of the safe-life approach is the a priori assumption that no damage is present on the structure during the entire service life: making such assumption will bring to a design where if a crack is present there is no inspection program defined to reveal such damage. For example, rotorcraft structures were designed with safe-life methodologies and they typically fail by unexpected fatigue cracking.

H.2.2 Fail safe design

The goal of this design philosophy is to build a structure that even if a component is cracked, failure would not produce the catastrophic lost of the complete structure. This design strategy is based on statically over-determined (hyperstatic) or redundant structures No regular monitoring in service is provided. In case of a fatigue failure, redistribution of forces provides an emergency life, so that the failure can be detected and repaired.

For fail-safe designs a fatigue analysis shall be performed for the elements of a multiple load path remaining after the rupture of one path by mean of Miner sum and crack propagation calculations.

When using Miner's sum (see 7.3 – route 1, 2 or 3), D_u is defined as the value which would be expected to cause the failure at the desired lifetime, for example:

$D_u = 0.5$ on a 20 years life time

When using crack propagation (see 7.3 – route 4), the failure criteria is given by a critical crack length, for example:

a_{cr} = 1/3 of a tube circumference

a_{cr} = crack length so that $K_I = K_{IC}$

For example, the aircraft industry tried to create wings and fuselages structures with a preferential crack path: the crack is forced to move on a predefined path by the use of crack arresters. They also have the advantage to prevent the cracks from propagating to a dangerous length before that the scheduled inspections will reveal the presence of the defect.

H.2.3 Damage tolerance design

A structure is defined damage tolerant when even in presence of damage due to corrosion, impact or fatigue, the operating life will not be affected: the damaged structure can sustain the loads without catastrophic failure before the damage would be found during the scheduled inspections. Regular monitoring in service is specified.

The fatigue fail-safe design concept has taken a step forward by assuming that the time needed to nucleate a crack is zero. The strategy is based on the assumption that a previously identified critical section contains a flaw or crack. So the design is based on the use of fracture mechanics to calculate the life cycles until failure From the number of life cycles, regular inspection intervals are derived. The dimension of such a flaw is taken as the biggest defect which cannot be identified by the needed inspection methods. Damage tolerance achieves the desired level of safety by using three distinct elements:

1. Damage limit: the maximum damage that the structure is able to sustain under limit load conditions.
2. Damage growth: the interval of damage propagation from the detection value to the damage limit.
3. Inspection program: a schedule of periodic inspections is set up to achieve timely detection of damage.

The maximum allowable damage that the structure can sustain at a critical fail-safe level is the key to the level of damage growth and inspection needed to ensure damage detection. The assessment is based on the residual strength diagram as a function of the crack length (Figure H. 2).

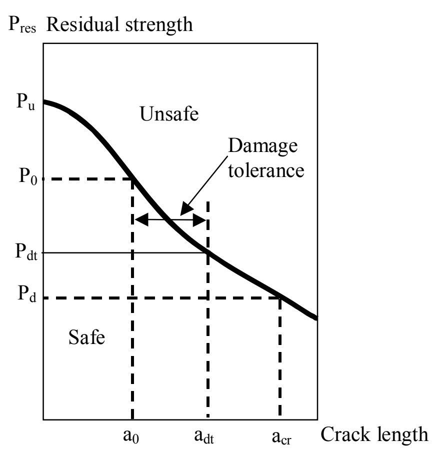

Figure H. 2: Residual strength diagram

The residual strength can be defined in terms of force, P_{res}, which the structure can still sustain. The relation between residual strength and crack length is:

$$P_{res} = f(a) \tag{H. 1}$$

where:

for an unaffected structure ($a=0$), the residual strength is equal to P_u, the ultimate load at which the construction fails because of plastic collapse.

for the critical crack length ($a=a_{cr}$), the load which the structure has to sustain once the needed safety factor s is applied, is equal to the maximum design load P_d which is higher that the "normal" operating load of the structure: $P_d = s\ P_u$

for a defect a_0 (missed by the employed inspection methods), the residual strength is equal to P_0

for the minimum value of the defect ($a=a_{dt}$) for which the component needs to be repaired or replaced, the residual strength is equal to P_{dt}

The corresponding dimension of the defect is a_{dt} , and therefore P_{dt} value, is set up with a satisfactory balance between production and maintenance costs and safety.

It is now possible to evaluate the number of cycles required to reach the maximum allowable dimension of the crack a_{dt}, starting from the initial value a_0 (see 7.3 - route 4).

H.3 Statistic and safety principle

The reliability analysis required that the random variable distributions are known. Practically the distribution characteristics used are the following:

- mean value
- standard deviation (Stdv)
- coefficient of variation (CoV)

But the real variable distribution of the "whole family" is neither known. Only we are able to obtained estimated distribution characteristics from samples of limited sizes. The estimated values are random variables from which safe values can be calculated when their distributions are known. Two different approaches exist, tolerance limits and prediction limits.

H.3.1 Mean, standard deviation estimation

Various formulae can be found to estimate the mean and standard deviation of a set of data [1]. The more commonly used ones are given here:

H.3.1.1 Mean

Measurements or test results are given in term of Y_i values versus fixed X_i values, such as number of cycles versus stress ranges.

When Y is a random variable for X fixed, the mean value of n values of Y is given by:

$$\widetilde{Y}_{mean} = \frac{\sum Y_i}{n} \tag{H. 2}$$

H.3.1.2 Standard deviation

When Y is a random variable for X fixed, the standard deviation s of n values of Y is given by:

$$(\widetilde{s})^2 = \frac{\sum (Y_i - Y_{mean})^2}{n-1} \tag{H. 3}$$

H.3.2 Tolerance limits

The safe value of a variable is obtained from its probability distribution by the determination of the limits between which it is stated that at least p % of the values lies with a confidence probability of γ. A tolerance limit can thus be regarded as a confidence limit on a confidence limit.

When the analysed parameter is normally distributed, the lowest expected real value is given by:

$$\mathbf{X}_{min} = \tilde{X} - k\tilde{s} \tag{H. 4}$$

where:

- $\mathbf{X}_{min}$ minimum expected value
- $\tilde{X}$ estimated mean value of X from the sample
- $\tilde{s}$ estimated standard deviation of X from the sample
- k coefficient corresponding to the selected exceeding probability and confidence level

The confidence level can be expressed by:

$$\text{prob}\{\tilde{X} - k\tilde{s} < X - Ks\} = \gamma \tag{H. 5}$$

where

- X real mean value of the "whole family"
- s real standard deviation value of the "whole family"
- K coefficient corresponding to the selected exceeding probability

To illustrate, for fatigue it is often considered for design that the safe level corresponds to a probability of survival (exceeding probability) of 95% with a confidence level of 75% [4]. In such case K corresponds to a survival probability for the "whole family" of 97.5 %.

ASTM [2] provides in tables of k versus:

- n number of data
- p exceeding probability in percent
- γ confidence level probability

Referring S-N curves, the safe design S-N curve constant C_D, using the ASTM table, is given by:

$$\log(C_D) = \log(C)_{mean} - k\, s_{logC} \tag{H. 6}$$

H.3.3 Prediction limits

The safe values of the mean and standard deviation are obtained by considering their confidence interval at a given probability level p, i.e., the interval of values in which the real value is expected to be with a probability equal to p.

H.3.3.1 Mean value, mean curve

It has been demonstrated that the distribution of the estimated mean value of a normally distributed variable can be determined from the following variable:

$$T = \sqrt{n}\,\frac{\tilde{m} - m}{\sqrt{\tilde{V}}} \tag{H. 7}$$

which follows a Student distribution [3] and where:

- m real mean value
- $\widetilde{m}$ estimated mean value
- $\widetilde{V}$ estimated variance
- n number of data

So the real value is, for a given probability of confidence γ, within the confidence interval defined as follow:

$$\widetilde{Y}_{mean} - t(\gamma, \mathrm{n}-1)\frac{\widetilde{s}}{\sqrt{n}} \le Y_{mean} \le \widetilde{Y}_{mean} + t(\gamma, \mathrm{n}-1)\frac{\widetilde{s}}{\sqrt{n}} \tag{H. 8}$$

where $t(\gamma,n-1)$ is the two sides Student distribution at (n-1) degrees of freedom and $\widetilde{s}$ the estimated standard deviation of Y.

When the relationship between Y and X is bi-dimensional, such as for the S-N curves, the formula is determined by a regression of the observed variable Y versus the different values of the fixed variable X. In case of the S-N curves, the regression log(N) function of log(ΔS), allows to determine the values of m and of the mean of log(C). In view of simplification, in many cases, the variable m is not considered random and the confidence interval is only determined and applied to log(C).

Note: *The regression has to be done log(N) function of log(ΔS) as the tests are performed with ΔS fixed and log(N) is the random variable. Doing the opposite, log(ΔS) function of log(N), may lead to erroneous values.*

H.3.3.2 Standard deviation

The distribution of an estimated standard deviation of a normally distributed variable can be determined from the following variable:

$$X = \frac{(n-1)\widetilde{V}}{V} \tag{H. 9}$$

which follows a chi-square distribution [3] and where:

- V real variance value
- $\widetilde{V}$ estimated variance
- n number of data

So the real value is, for a given probability of confidence γ, within the confidence interval defined as follow:

$$\frac{(n-1)\widetilde{s}^2}{\chi^2\left(\frac{1-\gamma}{2}, \mathrm{n}-1\right)} \le s^2 \le \frac{(n-1)\widetilde{s}^2}{\chi^2\left(\frac{1+\gamma}{2}, \mathrm{n}-1\right)} \tag{H. 10}$$

where $\chi^2(\gamma,n-1)$ is the chi-square distribution at (n-1) degrees of freedom.

H.3.3.3 Lowest expected value

When the analysed parameter is normally distributed, the lowest expected real value is given by:

$$\mathbf{X}_{\min} = \widetilde{\mathrm{X}} - \mathrm{k}\,\widetilde{\mathrm{s}} \tag{H. 11}$$

where:

$\mathbf{X}_{min}$ minimum expected value

$\tilde{X}$ estimated mean value of X

$\tilde{s}$ estimated standard deviation of X

k coefficient corresponding to the selected exceeding probability and confidence level

The value k corresponding to an acceptable risk level of probability (1-α), the minimum bound of the mean confidence interval and the maximum bound of the standard deviation confidence interval of probability γ is then given by:

$$k = \frac{t(\gamma, n-1)}{\sqrt{n}} + \Phi^{-1}(\alpha)\sqrt{\frac{n-1}{\chi^2\left(\frac{1+\gamma}{2}, n-1\right)}} \quad \text{(H. 12)}$$

where:

$t(\gamma, n-1)$ is the two sides Student distribution

$\chi^2(\beta, n-1)$ is the chi-square distribution at (n-1) degrees of freedom

$\Phi(\alpha)$ is the cumulative normal distribution

For S-N curve determination, the IIW (International Institute of Welding) [4] recommends α = 0.95 and γ = 0.75.

H.3.4 Coefficient of variation

In probabilistic approach another parameter is defined to characterize the random variable dispersion, the coefficient of variation V:

$$V = \frac{s}{m} \quad \text{(H. 13)}$$

where:

s standard deviation

m mean

When a random variable can be written, which is often the case [5]:

$$B = \Pi(B_i) \quad \text{(H. 14)}$$

the coefficient of variation of B is given by:

$$V_B = \sqrt{\Pi(1+V_{Bi}^2)-1} \quad \text{(H. 15)}$$

For example, if we consider the reference stress range ΔS_R at a hot spot, it can be written that:

$$\Delta S_R = B\, \Delta S_{determinist} \quad \text{and} \quad B = \Pi(B_i) \quad \text{(H. 16)}$$

and B is a random variable modelling all uncertainties occurring in the determination of stresses such as:

B_1 environmental and operational conditions

B_2 evaluation of the shape of the long term histogram

B_3 loads applied to the structure

B_4 modelling of the structure

B_5 workmanship

H.4 S-N curve assessment method (Routes 1,2 and 3)

The S-N curve assessment method corresponds to Routes 1, 2 and 3 (see 7.3).

H.4.1 Probabilistic approach

The probabilistic approach evaluates the probability that a structural detail has failed at the end of a specified service life time taking into account all the uncertainties and safety margin of the fatigue analysis. In the probabilistic approach, the cumulative fatigue calculation uses the mean values of extreme loads during the service time, the mean experimental S-N curve and the mean Miner sum D_u at failure. The risk of failure is expressed by a limit state function g, capacity minus demand so that:

if $g < 0$ the detail failed

if $g > 0$ the detail in safe

if $g = 0$ the detail is at the limit state

The limit state function can be written in various manner such as:

in terms of Miner sum $g = D_u - D$

in terms of life time $g = T - T_S$

where:

D_u capacity = Miner sum for failure

D demand = Miner sum resulting from the material strength and service conditions

T capacity = life time resulting from the material strength and service conditions

T_S demand = design service time

A probability distribution p_{Vi} is associated to each variable used for the stress of the histogram calculation (loads and dimensions) and p_{Du} to $\tilde{D}_u$ or p_{Ts} to T_S. In the following, we shall consider the limit state function expressed in terms of life time (T and T_S) [6]. The same procedure can be followed for the state function expressed in terms of a Miner's sum. The long term stress range histogram can be often defined by a 2 parameter Weibull law of shape factor ξ and characteristic value w [6]:

$$F(\Delta S) = \exp\left(-\left(\frac{\Delta S}{w}\right)^{\xi}\right) \tag{H. 17}$$

where:

$$w = \frac{\Delta S_R}{\left(-\ln p_R\right)^{1/\xi}} \tag{H. 18}$$

ΔS_R is the stress range at a probability p_R to be of exceeded.

The time at failure is then given by:

$$T = \frac{\left(-\ln(p_R)\right)^{m/\xi}}{f\ \Gamma\left(\frac{m}{\xi}+1\right)} \frac{C}{\Delta S_R^m} D_u \quad \text{(H. 19)}$$

where:

the S-N curve is given by: $\Delta S^m N = C$

f is the mean frequency of the cycles in Hz

D_u is the Miner sum at failure

Γ is the Gamma function

H.4.1.1 Failure probability direct calculation

Probability theory allows the combination of several probability distributions p_{Vi} to calculate the probability distribution P_T of T. Knowing the 2 probability distributions p_T and p_{Ts}, the probability of the risk of failure is given by:

$$prob\{T - T_S < 0\} = 1 - \int_0^\infty p_{Ts} dT_S \int_0^{Ts} p_T dT \quad \text{(H. 20)}$$

which is illustrated in Figure H. 3 by the hatched area.

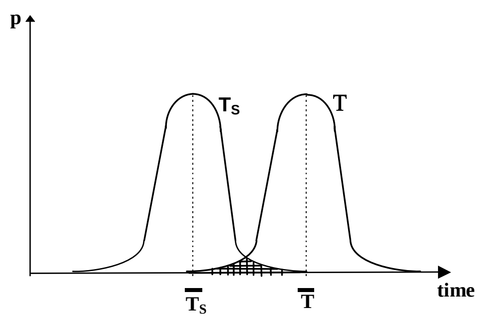

Figure H. 3 : Probability distributions and failure domain

Due to the need to combine the probability distributions of the variables entering in the stress range and Miner sum calculation, the determination of the probability of failure by integration is difficult and not really practicable during the design process. Therefore another way to calculate the failure probability has been developed based on the safety index.

H.4.1.2 Probability calculation using the safety index

When the limit state function g is normally distributed, the safety index β, when known, allows to calculated easily the failure probability by the following formula:

$$prob\{ g < 0 \} = \Phi(-\beta) \quad \text{(H. 21)}$$

where Φ is the cumulative normal distribution.

Two different safety index formulations have been developed: Cornel index, Hasofer-Lind index.

H.4.1.2.1 Cornel safety index

The Cornel safety index considers that the limit state function is expressed by the difference of 2 parameters, the capacity and the demand normally distributed. Considering the fatigue limit state function in terms of time, the Cornel safety index is given by:

$$\beta = \frac{(\overline{T} - \overline{T}_S)}{\sqrt{(s_T^2 + s_{Ts}^2)}} \tag{H. 22}$$

where:

$\overline{T}$, $\overline{T}_S$ mean value of T and T_S

s^2_T , s^2_{Ts} variance of T and T_S

The mean value of T is calculated in the same way than with the deterministic method, but using the mean values of the variables and the mean S-N curve, and not the design curve

Each variable with uncertainties used for the calculation of T is assumed normally or log-normally distributed and a coefficient of variation is defined which allows to calculate the standard variance s_T of T as given in H.3.4.

If the Cornell safety index is easy to calculated it presents some inconvenient:

- the coefficient is not robust versus the limit function format, for example, the two formula given in terms of T or D may not give the same value of β, all variables being the same.

- some variables cannot be considered normally distributed, in particular long term extreme values which are often exponential or Gumbell distributed.

H.4.1.2.2 Hasofer-Lind safety index

Due to the Cornel safety index inconvenient, the more commonly used safety index is the Hasofer-Lind index, defined as:

$$\beta = \min \sqrt{X^t X} \quad \text{for all } \mathrm{X} \in \mathrm{G}(\mathrm{X_i}) \tag{H. 23}$$

where:

X_i independent normalised random variables obtained from the original x_i variables by a Rosenblatt transformation

$G(X_i) = 0$ limit state function

This index can be geometrically illustrated as the minimum distance from the multidimensional space origin to the limit state surface as shown in Figure H. 4.

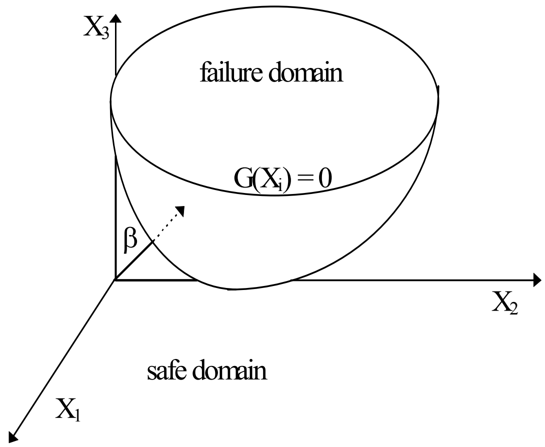

Figure H. 4: Hasofer-Lind safety index in a 3 dimensions space

For the fatigue assessment the limit state function G can be expressed by:

$$G = T - T_S \qquad \text{(H. 24)}$$

T and T_S being calculated as given previously.

This safety index is robust versus the limit state function format, but it requires the use of a specific software type FORM/SORM (First Order Moment / Second Order Moment), which is easy to obtain today.

H.4.2 Semi-probabilistic approach

The probabilistic approach, which defines the acceptable structure dimensioning through an acceptable probability of failure during the service time span, leads to a design implicit format procedure which is not practicable for a designer.

To solve this difficulty and maintain the probabilistic approach improvements, the semi-probabilistic format has been developed. It corresponds to a simple formulation similar to the deterministic method but with a clear identification of the safety margin attached to each variable entering in the fatigue assessment process [7]. This format allows the use of a level of safety attached to each random variable used in the cumulated fatigue calculation.

H.4.2.1 Semi-probabilistic format

The loads to be considered for the stress calculation are the mean values of loads during the service time increased by multiplication by a partial safety factor γ_i:

$$F_{design} = \gamma_i \, F_{mean} \qquad \text{(H. 25)}$$

The S-N curve is at a given distance below the mean experimental curve, i.e.:

$$\log(C_{design}) = \log(C_{mean}) - k_{SN} \, Stdv \qquad \text{(H. 26)}$$

where:

Stdv standard deviation of log(C)

k_{SN} safety margin in term of standard deviation of log(C)

The corresponding partial safety factor attached to the constant C is so: $\gamma_{SN} = 10^{(k_{SN} . Stdv)}$

The Miner sum is also affected by a partial safety factor γ_D (greater than 1):

$$D_{design} = \frac{D_{mean}}{\gamma_D} \tag{H. 27}$$

Then the fatigue assessment is performed following the same procedure than the deterministic approach as given in section 7.3.

Note: *In some standards, the given partial safety coefficient may be applied to an extreme value yet including a safety margin with respect to the mean value. It is generally the case for material strength for which standards define minimum guaranteed values.*

In the automotive industry it is often defined the 90%-driver which means that only 10% of the measurable spectra are higher than this one (let us call it "characteristic-spectrum"), which is supposed to enter the Miner-calculus. However, the stress ranges of the characteristic 90% design spectrum must be multiplied with the partial safety factor of the load side. In steel construction the used spectra for design have a return period of ~1week (Eurocode 1 & 3) which does not also correspond to a mean level.

H.4.2.2 Partial safety factor definition

The partial safety factor can be determined by the following formula:

$$\gamma = 1 + k\,V \tag{H. 28}$$

where V is the coefficient of variation of the considered parameter:

$$V = \frac{\text{standard deviation}}{\text{mean value}}$$

When the random variable is normally distributed, the probability to be greater for loads, lower for resistance, than the mean value is given by:

$p_i = 1 - \Phi(k_i)$ Φ: cumulative normal distribution

This formula allows to adjust the safety margin of each variable to:

- the selected safety level (97.5% of non failure, k = 1.96, 99.9% of non failure, k = 3.1)
- the level of knowledge of the uncertainty level of this variable by the value of V.

But the selection of the k and V values do not allow to directly obtain the selected global probability of failure of the structure. To do so it is necessary to calibrate the partial safety factors of all formula used to determine the structure component dimensions.

H.4.2.3 Partial safety factor calibration

The adjustment of the partial safety factors γ_i attached to the random variables to obtain the selected global probability of failure of the structure is performed by a calibration process [9] which requires 3 steps:

- definition of the code safety target index β^T, safety level to be ensured versus the application area
- calibration of the partial safety factors γ_i for each rule considered alone
- code calibration as a whole system, ensuring the coherence between the safety levels of all rules

The process is based on optimisation methods using the probabilistic approach (the dimensioning formula, the limit state function) and a penalty function with constraints.

The calibration process starts by the selection of:

- the target and minimum safety index (β^T , β_{min}),
- the acceptable ranges of the partial safety factors $(\gamma_i)_{min}$ and $(\gamma_i)_{max}$,
- a first set of partial safety factors $\{\gamma_i\}$ as defined in H.4.2.2.
- selection of the limit state functions G_P attached to the various dimensioning procedures
- selection of the penalty function P_f for the optimisation procedure.

Then an iteration is performed to determine the calibrated set of partial safety factors as follows:

1. calculation of the dimensions of the structure components using the code formula with the $\{\gamma_i\}$
2. calculation of the safety indexes β_p using a limit state function G_P and a reliability software such as FORM/SORM type
3. calculation of the penalty function P_f
4. verification of the optimisation (P_f minimum and constraints verified).

If the optimisation is not verified a new set of partial safety factors $\{\gamma_i\}$ is selected and the iteration is renewed from 1. The obtained set of $\{\gamma_i\}$ is considered acceptable when P_f is minimum, the constraints verified and the reliability indexes β_p as well as the most likely failure points relative to the several design cases are close to each others within 5% interval.

H.4.3 Random variable modelling

H.4.3.1 Probabilistic approach

For the probabilistic approach, the first step is to identify the parameters with uncertainties and to define their probability distribution. In general the load and strength variables are characterized by their mean, and the uncertainties, such as dimensions or modelling, are characterized by a bias and a variability expressed by a coefficient of variation. No standard can be considered existing yet as the probabilistic methods are used in R&D or rules development only. Example of data can be found in the marine field. An illustration is provided by the Bureau Veritas recommendation for steel ship fatigue analysis [4] and are given Table H. 1.

Parameter	Random variables	Distribution	Mean / Bias	V
Reference stress range $\Delta S_R = B\ \Delta S_{mean}$	ΔS: computed mean ref stress	lognormal	1.00	0.2 to 0.4
	$B = B_\eta\ B_H\ B_C\ B_W$		calculated	calculated
	B_η: Sea states description	lognormal	0.90	0.4 to 0.6
	B_H: Ship response on waves	"	0.85	0.1 to 0.3
	B_C: FEM stress computation	"	1.10	0.1 to 0.5
	B_W: Workmanship	"	0.90	0.1 to 0.3
Courbe S-N	K_p (1st slope – m = 3)	lognormal	1.342×10^{13}	0.438

2 slopes	K'_p (2nd slope – m = 5) N_q (change of slope)		$1.633*10^{17}$ 10^7	
Miner sum	B_{Du}	lognormal	1.0	0.3

Table H. 1: Random variables, distributions and characteristics

Always difficulties exist to fix the mean/bias and V values which is illustrated by the data of the Table H. 1 where for V limits are given instead of one value. In particular discussions are still important with respect to the Miner sum at failurethat varies strongly versus the load history irregularity [9]. Also other publications will show that other distribution laws can be acceptable, such as Gumbell for the ship response on waves instead of lognormal. The second step is to fix the acceptable probability level of safety. There is not yet enough return experience to give acceptable value in different industrial fields.

In marine, some applications have been performed on offshore platforms and ships [5], and with the generally accepted probability distributions in this industry area, the value of β lies between 1.5 and 3.

H.4.3.2 Semi-probabilistic format

For the semi-probabilistic approach, the first step is also to identify the parameters with uncertainties and for each of them provide the associated partial safety factors. The list of the uncertain parameters and the associated partial safety factors depends of the code or standard.

An illustration is given in Table H. 2 extracted from the Bureau Veritas rules for fatigue verification of steel ships [7].

Variables	Symbol	Values	
		General	Details at stiffener ends
Hull girder still water bending moment	γ_{S1}	1,00	1,00
Hull girder wave bending moment	γ_{W1}	1,05	1,15
Static water pressure	γ_{S2}	1,00	1,00
Wave water pressure	γ_{W2}	1,10	1,20
Strength	γ_R	1,02	1,02

Table H. 2: Partial safety factors for fatigue verification

These partial safety factors have been calibrated with respect to scantling with the previous rules for steel ship classification and return experience of inspections of ships with and without fatigue cracks.

H.5 Crack propagation assessment method (Route 4)

The crack propagation assessment method corresponds to Routes 4 (see 7.3).

H. 6 Applications in Fracture Assessment

The probabilistic procedure described in this section has been developed to calculate two different failure probabilities, PF:

(a) Probability of failure, defect size given by NDE.

(b) Probability of failure, defect not detected by NDE.

The procedure uses two different limit state functions, $g(X)$:

$$g_{FAD}(X) = g_{FAD}(K_{IC}, \sigma_y, a) = f_{FAD} - K_r \tag{H. 29}$$

$$g_{L_r}(\sigma_y, \sigma_U, a) = L_r^{\max} - L_r \tag{H. 30}$$

These limit state functions are based on the FITNET 'Known YS - Known UTS' continuous yielding FAD only (Option 1). Then, to calculate the probability of failure, a multi-dimensional integral has to be evaluated:

$$P_f = \Pr\left[g(X) < 0\right] = \int_{g(X)<0} f_x(x)dx \tag{H. 31}$$

$f_x(x)$ is a known joint probability density function of the random vector X. This integral is very hard (impossible) to evaluate, by numerical integration, if there are many random parameters.

H 6.1 Random parameters

Within the chosen procedure, the following parameters are treated as random parameters:

a) Fracture Toughness

b) Yield Strength

c) Ultimate Tensile Strength

d) Defect Size given by NDE

e) Defect not detected by NDE

f) Defect Distribution

These random parameters are treated as not being correlated with one another. The parameters can follow a normal, log-normal, Weibull or some special distributions (for the flaw size).

H6.1.1 Fracture toughness

The fracture toughness can follow a normal, log-normal or Weibull distribution. The normal probability density function has the following form:

$$f(K_I) = \frac{1}{\sigma_{K_{IC}}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left[\frac{K_I - \mu_{K_{IC}}}{\sigma_{K_{IC}}}\right]^2\right) \tag{H. 32}$$

where $\mu_{K_{IC}}$ is mean and $\sigma_{K_{IC}}$ is standard deviation.

The log-normal probability density function has the following form:

$$f(K_I) = \frac{1}{K_I \sigma_{\text{LogNor}} \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left[\frac{\ln(K_I) - \mu_{\text{LogNor}}}{\sigma_{\text{LogNor}}} \right]^2 \right) \tag{H. 33}$$

where μ_{LogNor} and σ_{LogNor} are the log-normal distribution parameters. They are related to the log-normal distribution parameters as follows:

$$\mu_{\text{LogNor}} = \ln(\mu_{K_{IC}}) - \frac{1}{2}(\sigma_{\text{LogNor}})^2 \tag{H. 34}$$

$$\sigma_{LogNor} = \sqrt{\ln\left[1 + \left(\frac{\sigma_{K_{IC}}}{\mu_{K_{IC}}} \right) \right]} \tag{H. 35}$$

The Weibull probability density function has the following form:

$$f(K_I) = \frac{k}{\theta} \left(\frac{K_I}{\theta} \right)^{k-1} \exp\left(-\left(\frac{K_I}{\theta} \right)^k \right) \tag{H. 36}$$

where θ and k are scale and shape parameters of the Weibull, respectively.

The normal distribution parameters $\mu_{K_{IC}}$ and $\sigma_{K_{IC}}$ are related to the Weibull distribution parameters as follows:

$$\mu_{K_{IC}} = \frac{\theta}{k} \Gamma\left(\tfrac{1}{k}\right) \tag{H. 37}$$

$$\sigma_{K_{IC}} = \sqrt{\frac{\theta^2}{k} \left[2\Gamma\left(\tfrac{2}{k}\right) - \frac{1}{k} \Gamma\left(\tfrac{1}{k}\right)^2 \right]} \tag{H. 38}$$

where Γ(z) is the gamma function, defined by the integral:

$$\Gamma(z) = \int_0^\infty t^{z-1}.e^{-t} dt \tag{H. 39}$$

This non-linear system of equations (H. 37)-(H. 38) is solved using a globally convergent method with line search and an approximate Jacobian matrix.

H.6.1..2 Yield Strength and Ultimate Tensile Strength

The Yield strength and Ultimate tensile strength can follow a normal, log-normal or Weibull distribution. These are accorded the same treatment as fracture toughness data discussed in 0.

H6.1.3 Defect size given by NDE

The defect size given by NDE can follow a normal, log-normal or exponential distribution. For the treatment of data and distribution parameters, using a normal a log-normal distribution, see Section H.3.1 above.

The exponential probability density function has the following form:

$$f(a) = \lambda \exp(-\lambda a) \quad \text{(H. 40)}$$

where λ is the exponential distribution parameter. The mean value μ_a (equal to the standard deviation, σ_a, for this distribution) and is related to λ as follows:

$$\mu_a = \sigma_a = \frac{1}{\lambda} \quad \text{(H. 41)}$$

H6.1.4 Defect Not Detected by NDE

The defect not detected by NDE is treated as a deterministic parameter in the analysis.

H6.1.5 Defect Distribution

The defects can follow a normal, log-normal, exponential distribution and accorded the same treatment as described in Section H.3.1.

H6.2 Calculation of Failure Probabilities

As mentioned in Section 0, the failure probability integral (H. 31) is very hard to solve using numerical integration. Instead, the following numerical algorithms are included within the procedure:

- Simple Monte Carlo simulation
- First-order reliability method (FORM)

H6.2.1 Simple Monte Carlo Simulation

MCS is a simple method that uses the fact that the failure probability integral can be interpreted as a mean value in a stochastic experiment. An estimate is therefore given by averaging a suitably large number of independent outcomes (simulations) of this experiment.

The basic building block of this sampling is the generation of random numbers from a uniform distribution (between 0 and 1). A simple algorithm will repeat itself after approximately 2.10^3 to 2.10^9 simulations and is therefore not suitable to calculate medium to small failure probabilities. Once a random number, u, between 0 and 1, has been generated, it can be used to generate a value of the desired random variable with a given distribution. A common method is the inverse transform method. Using the cumulative distribution function $F_x(x)$, the random variable would then be given as:

$$x = F_X^{-1}(u) \quad \text{(H. 42)}$$

To calculate the failure probability, one performs N deterministic simulations and for every simulation checks if the component analysed has failed (i.e. if g(X) < 0). The number of failures are NF, and an estimate of the mean probability of failure is:

$$P_{F,MCS} = \frac{N_F}{N} \quad \text{(H. 43)}$$

An advantage with MCS, is that it is robust and easy to implement into a computer program, and for a sample size $N \to \infty$, the estimated probability converges to the exact result. Another advantage is that MCS works with any distribution of the random variables and there are no restrictions on the limit state functions.

However, MCS is rather inefficient, when calculating failure probabilities, since most of the contribution to PF is in a limited part of the integration interval.

H6.2..2 First-Order Reliability Method (FORM)

FORM uses a combination of both analytical and approximate methods, when estimating the probability of failure. First, one transforms all the variables into equivalent normal variables in standard normal space (i.e. with mean = 0 and standard deviation = 1). This means that the original limit state surface g(x) = 0 then becomes mapped onto the new limit state surface gU(u) = 0.

Secondly, one calculates the shortest distance between the origin and the limit state surface (in a transformed standard normal space U). The answer is a point on this surface, and it is called the most probable point of failure (MPP), design point or β-point. The distance between the origin and the MPP is called the reliability index β_{HL}. In general, this requires an appropriate non-linear optimisation algorithm.

Then one calculates the failure probability using an approximation of the limit state surface at the most probable point of failure. Using FORM, the surface is approximated to a hyperplane (a first order/linear approximation).

The probability of failure is given as:

$$P_{F,FORM} = \Pr\left[g_{Linear}(u) < 0\right] = \Phi(-\beta_{HL}) \tag{H. 44}$$

$$P_{F,SORM} = \Pr\left[g_{Quadratic}(u) < 0\right] \approx \Phi(-\beta_{HL}) \prod_{i=1}^{N-1} \left(1 - \kappa_i \beta_{HL}\right)^{-1/2} \tag{H. 45}$$

Φ(u) is the cumulative distribution function in standard normal space and κi is the principal curvature of the limit state surface at the most probable point of failure (MPP). FORM is more computationally efficient compared to MCS. Using the implementation within FITNET, quite accurate results can be obtained for failure probabilities between 10^{-1} to 10^{-15}. A disadvantage is that the random parameters must be continuous, and every limit state function must also be continuous.

H6.3 Semi-probabilistic approach

Partial safety factors are factors which can be applied to the individual input variables in a design equation to give the given target reliability without having to carry out full probabilistic calculations.

H6.3.1 Partial safety factors determination

The overall partial safety factor for load effects is the ratio of the design point value to the value assumed to represent the loading, and the overall partial safety factor on resistance effects is the ratio of the value chosen to represent resistance effects to the design point value. However, no unique solution for partial safety factors exist and the same target reliability level can be achieved by different combinations of factors.

H6.3..2 Recommended values of partial safety factors

The partial safety factors to be applied in assessments depend both on the target reliability required and on the scatter or uncertainty of the main input data, namely fracture toughness, stress level, flaw size and yield strength. Partial safety factors for given target reliabilities and different degrees of variability of the input data are given in this part of the procedure. The target reliability levels chosen correspond to the four conditions defined in Table H. 3 and an additional high reliability level, corresponding to a failure probability of 10^{-7}, representative of very high structural integrity requirements as would be applied to highly critical components. The failure probabilities of 0.23, 10^{-3}, $7\text{x}10^{-5}$, 10^{-5} and 10^{-7} correspond to target reliability index values of β = 0.739, 3.09, 3.8, 4.27 and 5.2, respectively.

Failure consequences	Redundant Component	Non-redundant Component
Moderate	2.3×10^{-1}	10^{-3}
Severe	10^{-3}	7×10^{-5}

Table H. 3: Target failure probability

Partial safety factors to achieve the required reliability have been derived using first order second moment reliability analysis methods for different coefficients of variation of stresses, flaw sizes, fracture toughness and yield strength. For stress levels, coefficients of variation of 0.1, 0.2 and 0.3 with a normal distribution are considered with a COV (Co-efficient of Variation = standard deviation/ mean) of 0.2 representing dead load or residual stress effects and a COV of 0.3 representing live load effects. For the purposes of determining partial safety factors the results are derived in terms of different COV values so that for application purposes it is necessary to know both the best estimate (mean) value of defect size and the standard deviation to determine the appropriate COV. Weibull and lognormal distributions were adopted for fracture toughness data with coefficients of variation of 0.2 and 0.3 and a lognormal distribution for yield strength with a coefficient of variation of 0.10.

The resulting recommendations for partial safety factors to be applied to the best estimate (mean) values of maximum tensile stresses and flaw sizes, and to the characteristic (i.e. minimum specified) value of toughness and yield strength, are given in Table H. 4 It should be noted that the partial safety factors on fracture toughness are applicable to mean minus one standard deviation values as an approximate estimate of lowest of three. It is recommended that sufficient fracture toughness tests should be carried out to enable the distribution and mean minus one standard deviation to be estimated satisfactorily.

Partial factors on yield strength have little effect other than at high L_r values when plastic collapse is the dominant mechanism and hence the material factors already in use for EuroCode 3 on yield strength are adopted for consistency. For partial safety factors on stress, the values for β = 3.8 are chosen as 1.35 and 1.5 for stress COVs of 0.2 and 0.3 to represent dead and live load respectively, and to be consistent with EuroCode 3. It must be recognised that the partial safety factors will not always give the exact target reliability indicated but should not give a probability of failure higher than the target value. The recommended partial safety factors give 'safe' results for the target reliability over the whole range.

The analyses and recommendations given above are based on the assumption that failure will occur when an assessed defect gives rise to a point which falls on the failure assessment diagram, whereas, in practice it is often found that the diagram gives safe predictions rather than critical ones. Including these modelling uncertainties in the calculations of partial factors will lead to a modified set of factors. However, it is not intended that these modified factors be used for general safety assessments and since further work is required prior to their implementation they are not covered in any further detail here.

		p(F) 2.3×10^{-1}	p(F) 10^{-3}	p(F) 7×10^{-5}	p(F) 10^{-5}	p(F) 10^{-7}
		β = 0.739	β = 3.09	β = 3.8	β = 4.27	β = 5.2
Stress	$(COV)_\sigma$	γ_σ	γ_σ	γ_σ	γ_σ	γ_σ
Extreme	01	1.05	1.2	1.25	1.3	1.4
Dead+Res	0.2	1.1	1.25	1.35	1.4	1.55
Live	0.3	1.12	1.4	1.5	1.6	1.8
Flaw size	$(COV)_a$	γ_a	γ_a	γ_a	γ_a	γ_a
	0.1	1.0	1.4	1.5	1.7	2.1
	0.2	1.05	1.45	1.55	1.8	2.2

	0.3	1.08	1.5	1.65	1.9	2.3
	0.5	1.15	1.7	1.85	2.1	2.5
Toughness, K	$(COV)_K$	γ_K	γ_K	γ_K	γ_K	γ_K
	0.1	1	1.3	1.5	1.7	2.0
(min of 3)	0.2	1	1.8	2.6	3.2	5.5
	0.3	1	2.85	NP	NP	NP
Yield strength	$(COV)_M$	γ_M	γ_M	γ_M	γ_M	γ_M
(on min spec.)	0.1	1	1.05	1.1	1.2	1.5

Table H. 4: Recommended partial factors for different combinations of target reliability and variability of input data based on failure on the FAD.

Notes:
γ_σ is a multiplier to the mean stress of a normal distribution
γ_a is a multiplier to the mean flaw height of a normal distribution
γ_K is a divider to the mean minus one standard deviation value of fracture toughness of a Weibull distribution
γ_M is a divider to the mean minus two standard deviation value of yield strength of a log-normal distribution

H 7 Inspection programme

Timely detection of damage is the ultimate control in guarantying structural integrity. From an economical point of view the maintenance inspections are costly and time consuming: the trend of the industry is to look for a design solution which permits to expand the *N* value, also called time of damage *H*; commonly the time of inspections, *I*, is set at the half of the damage time.

The underdevelopment RBI (Risk Based Inspection) approach allows to optimised the inspection periodicity using the probabilistic approach.

H7.1 RBI principle

The RBI principle is to determine the probability of crack and structure loss function of operation time, to define an inspection procedure, a strategy when a fatigue crack is observed during an inspection (to do nothing, to repair, to replace the component), to calculate the costs of the inspection, the repair, the replacement, the structure loss and to optimise the total cost on the structure life design time [10], [11].

With respect to fatigue the failure probability can be expressed using the safety index β versus time:

$$\beta(t) = \frac{\ln(\overline{T}) - \ln(t)}{s_{\ln(T)}} \qquad \text{(H. 46)}$$

where:

$\overline{T}$ mean life time obtained from the fatigue assessment

t passing time

$s_{\ln(T)}$ standard deviation of ln(T)

Fixing an upper acceptable limit of probability of failure p_{max}, inspection is due when prob = $\Phi(-\beta)$ reaches this upper limit (Figure H. 5)

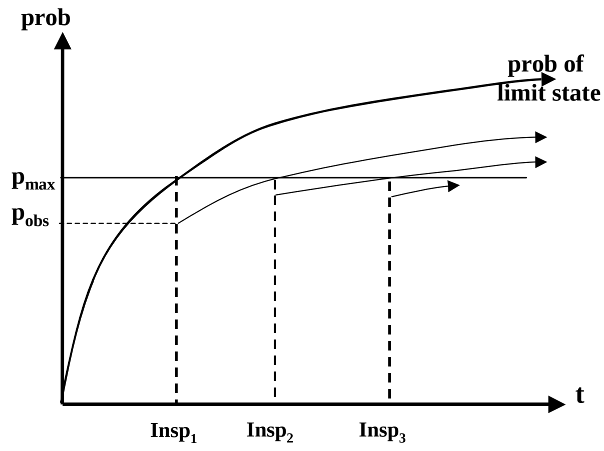

Figure H. 5: Failure probability versus time and due inspections

When an inspection is performed, the ratio of observed failures divided by the number of verified details provides an estimation of the real probability of failure which allows to adjust the cumulative fatigue calculation assumptions and method and the β calculation assumption and procedure. After adjustment, a new curve of probability of failure versus time can be determined from which the next inspection time can be fixed (Figure H. 5). The optimisation is obtained by selecting the maximum acceptable probability of failure, the inspection procedure (inspection method accuracy), and the decision criteria of doing nothing, repairing, replacing components. Total cost is determined from the inspection, repair, replacement, structure loss costs and a repair, survival and failure event tree.

H. 7.2 Maintenance interval increase strategy

The maintenance intervals are interspersed to guarantee the safety of the structure and to get the most of the damage tolerance of the material. To obtain such result it is necessary to increase the time of damage *H*. A way is to improve the inspection procedures in order to decrease the detectable dimension of the defect, a_d (Figure H. 6).

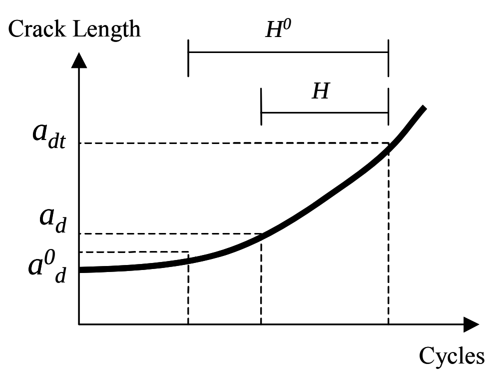

Figure H. 6: Time of damage versus the detectable defect dimension a_d

Decreasing the detectable defect size a_d to a_d^0, the allowable time of damage *H* becomes H^*. Another way to increase *H* is to use redundant structures or arresters. Those structures will make possible respectively the creation of multiple load paths and the stop of the crack growth. By using arresters a bigger defect dimension is permitted. An illustration is given in Figure H. 7

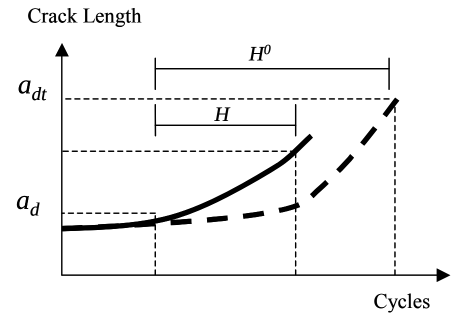

Figure H. 7: Time of damage *H* after inserting arresters

The last solution is to improve the damage tolerance of the material as illustrated in Figure H. 8.

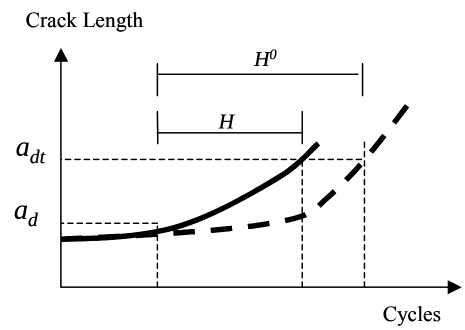

Figure H. 8: Time of damage *H* by optimising the damage tolerance

The life prediction of a component, obtained with the damage tolerant methodology, could be affected by several factors. The crack length at the inspection time could be different from the crack length calculated with fracture mechanics principles due to many uncertainties still present. In particular we shall mention the dimension of the initial and of the final crack length, the proper definition of the loading spectrum, the material data used to obtain the growth rate, the method of integration of such data and the inspection technology.

H 8 Application in the Creep Regime

H 8.1 Determination of creep rupture life

The main input data required for assessing structures prone to creep damage are the rupture time, defined as a function of the temperature T and stress σ. Experiments indicate that the rupture time shows significant scatter and hence it should be considered as a random variable.

A lognormal distribution can be selected to describe the randomness of the rupture time as follows:

$$\ln \tau = \mu_c + \delta_c \Omega \ , \qquad \text{(H. 47)}$$

where τ is the random variable - rupture time, Ω is the normalized Gaussian distribution function N(0;1) and μ_c and δ_c are the mean value and standard deviation of the rupture time, expressed in the logarithmic base, respectively.

The total accumulated creep damage D_c can be obtained using the life fraction rule. Assuming that the operation of the investigated structure is broken into a series of blocks $[\sigma_i;T_i]$ during which the load/stress σ_i and the temperature T_i are sensibly constant , the total accumulated damage is then given by the following equation:

$$D_c = \sum_{i=1}^{n} \frac{\Delta t_i}{\tau_i(\sigma_i;T_i)}, \tag{H. 48}$$

where Δt_i is the time during which the structure is subjected to the stress σ_i and temperature T_i. Values τ_i are given by equation (H.47). Substituting equation (H.47) into equation (H.48) and introducing the relation $t_{ri} = \exp[\mu_c(\sigma_i;T_i)]$ lead to:

$$D_c = \exp(-\delta_c\Omega)\sum_{i=1}^{n} \frac{\Delta t_i}{t_{ri}}. \tag{H. 49}$$

The total creep damage accumulation D_c must be less than unity ($D_c < 1$) for rupture not to occur within the investigated structure. The probability that D_c is less than 1 can be expressed as follows:

$$P(D_c < 1) = P(\ln D_c < 0) = P\left(-\delta_c\Omega + \sum_{i=1}^{n}\frac{\Delta t_i}{t_{ri}} < 0\right) = P\left(\Omega > \frac{1}{\delta_c}\sum_{i=1}^{n}\frac{\Delta t_i}{t_{ri}}\right) = 1 - N(Z), \tag{H.50}$$

where N is a normalized Gaussian distribution function and Z is defined by the following equation:

$$Z = \frac{1}{\delta_c}\sum_{i=1}^{n}\frac{\Delta t_i}{t_{ri}}. \tag{H. 51}$$

As the creep damage cannot exceed a value of 1, the probability $P(D_c \geq 1)$ reduces to the probability $P(D_c = 1)$ and hence using equation (H.50), the probability of rupture is given by:

$$P(D_c = 1) = N(\Omega). \tag{H. 52}$$

H 8.2 Stochastic creep crack growth

In the case of a cracked structure operating in the creep regime, the creep crack growth rate can be correlated satisfactorily in terms of the creep fracture parameter *C** using the following relation:

$$\frac{da}{dt} = A_c(C^*)^{n_c}. \tag{H. 53}$$

Material parameters A_c and n_c can both be considered as random variables. However, experimental results show that the parameter n_c does not exhibit significant scatter and hence it is generally considered as a constant. The scatter in the random variable A_c is usually described by a lognormal distribution as follows:

$$\ln A_c = \mu_A + \delta_A\Omega, \tag{H. 54}$$

where Ω is the normalized Gaussian distribution function N(0;1) and μ_A and δ_A is are the mean value and standard deviation of the logarithm value of A_c, respectively.

When several of the inputs to the creep crack growth assessment are considered as random variables (e.g. initial crack size, creep rupture time, creep crack growth constant A_c), the determination of the distribution function of the crack size becomes a complex mathematical expression that is hard to evaluate using numerical integration. In this case, the use of suitable numerical methods, such as Monte Carlo techniques, is required. However, if a sensitivity analysis of the key inputs to the creep crack growth assessment shows that the scatter in the creep crack growth constant A_c has a dominant effect, then the other input data can be assumed constant and a simplified procedure can be employed. In this case, equation (H.53) can be integrated numerically and the crack size $a(t_1)$ at time t_1 can be derived as follows:

$$a(t_1) = \int_0^{t_1} A_c (C^*)^{n_c}\, dt = \exp(\delta_A \Omega) \int_0^{t_1} A_c^{av} (C^*)^{n_c}\, dt = \exp(\delta_A \Omega) \cdot a^{av}(t_1)\,, \qquad \text{(H. 55)}$$

where

$$A_c^{av} = \exp(\mu_A), \quad a^{av}(t_1) = \int_0^{t_1} A_c^{av} (C^*)^{n_c}\, dt\,. \qquad \text{(H. 56)}$$

The probability of the crack size $a(t_1)$ being less than a given crack size a_f is given by:

$$P(a \le a_f) = P(\ln a \le \ln a_f) = P\left(\Omega \le \frac{\ln a_f - \ln[a(t_1)]}{\delta_A} \right) = N(Z_A)\,, \qquad \text{(H. 57)}$$

where the random variable Z_A has the normalized Gaussian distribution function N(0;1) and is defined by the following relation:

$$Z_A = \frac{\ln a_f - \ln[a(t_1)]}{\delta_A}\,. \qquad \text{(H. 58)}$$

Equation (H.57) defines the distribution function of the crack size at time t_1. If the calculation is performed for different values of time t_1 then the distribution function of crack size becomes a function of time. Furthermore, the probability of exceeding the crack size a_f is $P(a > a_f) = 1 - N(Z_A)$. If a_f is selected as the critical crack size, the probability of fracture is given by the value $1 - N(Z_A)$.

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